On a non-local parabolic free boundary problem arising in a theory of financial bubbles

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In this talk, we study a non-local free boundary problem arising in financial bubbles. The model equation, studied here, is the following free boundary problem formulated as a Hamilton-Jacobi equation:

$$\min(Lu, u(t,x) - u(t,-x) - \psi(t,x)) = 0, \quad (t,x) \in \mathbb{R}^+ \times \Omega, \qquad (1)$$

where $\Omega \subset \mathbb{R}$ is a symmetric bounded domain such that if $x \in \Omega$ then $-x \in \Omega$ and $\psi \in C^2(\mathbb{R}^+ \times \Omega)$, and the operator L is the following parabolic operator

$$Lu = u_t + a^{ij}(x) D_{ij} u + b^i(x) D_i u + c(x) u, \qquad a^{i,j} = a^{j,i}.$$

Here the coefficients $a^{i,j}$, b^i, c are assumed to be continues and the matrix $[a^{i,j}(x)]$ is positive definite for all $x \in \Omega$. Additionally we assume that the coefficients are "symmetric" in the domain Ω.

We discuss iterative method for numerical results, which consists of a sequence of parabolic obstacle problems at each step to be solved, that in turn gives the next obstacle function in the iteration. The convergence of the proposed algorithm is proved. We study the finite difference scheme for this algorithm and obtain its convergence.

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