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Contents

Video-lecture courses: development experience and ways to improve efficiency

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In recent years, teachers of the Vorovich Institute of Mechanics, Mathematics, and Computer Science (IMMCS) of the Southern Federal University have developed video-lectures related both to some topics [1], and to the total educational courses [2]. In particular, the author of this report recorded in the 2015-2016 and 2018-2019 academic years a series of 82 video-lectures, which covers a four-semester course "Calculus" ("Mathematical Analysis"). Lectures are posted on youtube.com, and are also available from the IMMCS website (edu.mmcs.sfedu.ru, the "Video-lectures" section). According to statistics provided on youtube.com, the number of lecture views by August 2019 was 89 thousand, and the total viewing time was 524 thousand minutes. The report discusses ways to improve the effectiveness of video-lectures and increase audience. One of such ways is the preparation of subtitles in both Russian and English, which, in particular, will make it possible to use video-lectures in English-language educational programs.

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On a semilattice formed by a special binary relation over the set of regular languages

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For the description of a regular language, there are different invariants: not only well-known canonical automata, but universal automata [1], basis automata [2], complete automata [3]. More precisely, each of these invariants could be called a complete invariant.

In considering both the basis and universal automata, we need to construct canonical au-tomata both for the given regular language L and for its mirror image L^R. In the process of such a construction, we get, among other things, a special binary relation given on the state pairs of these two canonical automata; according to our terminology, this is the relation #. This relation is also an invariant of the regular language (however, it is not its complete invariant).

For every binary relation #, there exists a whole subclass of the class of regular languages with this relation #. On the set of all regular languages we can define a binary relation (let R) that holds for some two languages if and only if they get the same binary relation #. The binary relation R defined in this way on the set of all regular languages is an equivalence relation. Thus, the question arises of the "most typical" language, which could be called a representative of the equivalence class with respect to R. In this paper, we describe languages that can be considered like the "typical representatives" of such equivalence classes. The subclasses of binary relations defined by us form a semilattice.

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On the solution of the fractional Bratu's problem: Laplace decomposition approach

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The ultimate aim of the article is to apply an efficient semi-analytical method for the approximate solution of the fractional differential equations of Bratu type. A Laplace Decomposition Method (LDM), which is a combination of Laplace transform and a Decomposition Method, is implemented for the nonlinear fractional Bratu problem that is complemented with initial and boundary conditions. The nonlinear term is decomposed and a recursive algorithm is composed for the determination of the proposed infinite series solution. Some examples are selected to explicate the effectiveness and simplicity of the proposed technique. The results assure that this scheme is speedily convergent and quite accurate by which it approximates the solution using only few iterates of its iterative scheme.

On homeomorphism between Euclidean subspace and conformally Euclidean manifold

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As A. Poincare rightly pointed out, there is no finest geometry, there is a geometry convenient for solving a specific task. Usually, when studying complex dynamical systems, coordinate transformations are used to separate variables and reduce the original system. In particular, by coordinate transformations, the three-body problem, which is a system of 18th order, is reduced to the system of 8th order. However, as we show, it is possible to make the reduction of a dynamical system more complete if we use the curve (Riemannian) geometry. Note that in this case it becomes possible to reveal the hidden symmetries of internal motion and, accordingly, to obtain additional integrals of motion. For a three-body system, replacing the geometry allows us to reduce the problem to the 6th order system. The main difficulty arising at the solution of this problem is the generalization of the well-known Poincare theorem on a homomorphism between the 3D sphere with unit radius and 3D compact. In this work, the possibility of such a generalization is strictly proved.

Recent trends and future directions of the polycirculant conjecture

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Fixed-point-free permutations, also known as derangements, have been studied for centuries. In 1981, Marusic asked whether there exists a vertex-transitive graph having a fixed-point-free automorphism of prime order. This talk is devoted to surveying results, open problems and methods in this area.

Quantum Fluid Dynamics Representation of the Schrodinger, Nonlinear Schrodinger and the Gross-Pitaevskii equations with Navier-Stokes dissipation

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The Quantum Fluid Dynamics (QFD) representation has its foundations in the works of Madelung (1929), De Broglie (1930 - 1950) and Bohm (1950 - 1970). QFD consists of a set of nonlinear Partial Differential Equations, although it is derived from and is equivalent to the Schrodinger equation that is linear. This approach is partly motivated by Einstein's questioning of the completeness of quantum mechanics as he thought the complete theory had to include nonlinearities. This paper extends the QFD formalism of quantum mechanics to include dissipation.

The Navier - Stokes type of dissipation is a phenomenological description of random oscillations in the fluid velocities through collisions with their environment.

The signi cance of the Navier Stokes type of dissipation is in its being the basic form that keeps the invariance of the equations under Galilean transformation. The nonlinear field equations with the dissipation transform the QFD differential equations from hyperbolic to parabolic type, thereby opening a natural framework for studying fundamentally nonlinear phenomena. The formulation presented here conserves probability while the momentum and energy equations lead in a natural way to decay, vorticity, de-phasing and possibly chaos.

Two numerical examples illustrate the impact of dissipation: (i) tunneling through and reflection by an energy barrier as a model chemical reaction; and (ii) the harmonic excitation of an harmonic oscillator. In (ii), the unreal theoretical singularities of the amplitudes at resonances are reduced to finite values. In (i), the diffusion brought in by the parabolic character of the PDE's enhances the quantum tunneling.

The same representation of QFD applies to the Gross-Pitaevskii equation and the Nonlinear Schrodinger equation.

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On Bergman type operators on Lipschitz and mixed norm spaces over the real ball

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In the setting of the unit ball B in $\mathbb{R}^n$ $(n \geq 2)$, we define two families of Bergman type operators $\Phi_{\beta,\delta}$ and $T_{\beta,\lambda}$ constructed with ordinary harmonic Poisson kernel in B and a special harmonic Poisson-Bergman type kernel.

We prove the operator $\Phi_{\beta,\delta}$ continuously maps the Lipschitz space Λ_α onto harmonic Lipschitz space $h\Lambda_{\alpha+\beta-\delta}$. In particular, for $\beta = \delta$, the operator $\Phi_{\beta,\beta}$ is a continuous projection of the Lipschitz space Λ_α onto its harmonic subspace. Also, the operators $T_{\beta,\lambda}$ continuously map mixed norm spaces $L(p, q, \alpha)$ into itself for certain values of parameters. For a special choice of parameters, the operators continuously project mixed norm spaces $L(p, q, \alpha)$ onto their harmonic subspaces. Corresponding norm inequalities are stated.

Evolution of the first eigenvalue of the geometric operator on manifold along the geometric flow

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We investigate the evolution, monotonicity for the first nonzero eigenvalue of the geometric operator on closed Riemannian manifold along the geometric flow. We show that the first nonzero eigenvalue is nondecreasing under the geometric flow under certain geometric conditions and find some applications in low dimensional manifolds.

On n-pseudo valuation domains

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Let n be a positive integer and R be an integral domain with quotient field K. A prime ideal P of R is called an n-powerful semiprimary ideal of R if whenever $x^n y^n$ in P for some x, y in K, then x^n in P or y^n in P. If every prime ideal of R is an n-powerful semiprimary ideal of R, then R is called an n-pseudo valuation domain (n-PVD). A number of results and examples will be presented. Note that if n is 1, then R is a 1-PVD if and only if R is a PVD in the sense of Hedstrom-Houston.

Mathematical modeling of the semi-Markovian random walk processes with jumps and delaying screen by means of a fractional order differential equation

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In this talk, we reduce the semi-Markovian random walk processes with jumps and delaying screen in zero. The Laplace transform on time, Laplace-Stieltjes transform on phase of the conditional distribution of semi-Markovian random walk processes with jumps is given. We get a mathematical modeling of the semi-Markov random walk processes with a delaying screen in zero, given in general form by means of integral equation. In this talk, the residence time of the system is given by the gamma distribution with the parameters α and β resulting in a fractional order integral equation. The purpose of this talk is to reduce the fractional order integral equation to a fractional order differential equation. Finally, we find the exact solution of fractional order differential equation.

Split metric decomposition of finite metric spaces via Gromov products structures

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Let (X, d) be a finite metric space with elements $P_i, i = 1, \ldots, n$ and with distance functions $d(P_i, P_j) =: d_{ij}$ for $i, j = 1, \ldots, n$. The "Gromov product" Δ_{ijk}, is defined as $\Delta_{ijk} = 1/2(d_{ij} + d_{ik} - d_{jk})$. (X, d) is called Δ-generic, if, for each fixed i, the set of Gromov products has a unique smallest element, $\Delta_{ij_ik_i}$. The Gromov product structure on a Δ-generic finite metric space (X, d) is the map that assigns the edge $P_{j_i}P_{k_i}$ to P_i. Two metric spaces (X, d) and (X, d') are said to be Δ-equivalent, if the corresponding Gromov product structures are the same up to a permutation of X. A finite metric space is called "quadrangle generic", if for all 4-point subsets $\{P_i, P_j, P_k, P_l\}$, the set $\{d_{ij} + d_{kl}, d_{ik} + d_{jl}, d_{il} + d_{jk}\}$ has a unique largest element. We define the "quadrangle structure" on a quadrangle generic finite metric space (X, d) as the map that assigns to each 4-point subset of X the pair of edges corresponding to the maximal element of the sums of distances. Two metric spaces (X, d) and (X, d') are said to be Q-equivalent, if the corresponding quadrangle structures are the same up to a permutation of X. For $n = 4$ and $n = 5$, Δ-equivalence coincides with Q-equivalence. For $n \geq 6$, there are in general more than one quadrangle structures compatible with a given Gromov product structure. We prove that, (i) The isolation index of the 1-split metric δ_i is equal to the minimal Gromov product at the vertex P_i; (ii) The isolation index of the 2-split metric δ_{ij} is nonzero if and only if the edge P_iP_j is a side in every quadrangle whose set of vertices includes P_i and P_j; (iii) The isolation index of a k-split metric $\delta_{i_1 \ldots i_k}$ is nonzero if and only the edge $P_{i_m}P_{i_n}$ is a side in every quadrangle whose vertex set contains P_j only for $j = i_m$ and $j = i_n$.

Some structural results of certain nuclear Frechet spaces

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Power series spaces constitute an important and well-studied class in the theory of Frechet spaces. The diametral dimension of a nuclear Frechet space with the properties weak-DN and Omega is related to some power series spaces of infinite type and finite type. In recent years, significant structural results were obtained for the class of nuclear Frechet spaces with the properties weak-DN and Omega whose diametral dimension coincides with that of a power series space. We first introduce the invariants which are mentioned above and the aim of this talk is to give some structural results about the structure of nuclear Frechets space with the properties weak-DN and Omega.

Shell equation derived by a Γ-limit

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We consider boundary value problems (BVPs) of bending elastic isotropic thin media $\Omega^h : \mathcal{C} \times [-h, h]$ around a surface $\mathcal{C}$ with the Lipshitz boundary $\Gamma := \partial\mathcal{C}$, governed by the Láme equation

$$\begin{aligned} &\mathcal{L}_{\Omega^h}\mathbf{U}(x) = \mathbf{F}(x), && x \in \Omega^h := \mathcal{C} \times (-h, h), \\ &\mathbf{U}^+(t) = 0, && t \in \Gamma_L^h := \partial\mathcal{C} \times (-h, h), \\ &(\mathfrak{T}(t, \nabla)\mathbf{U})^+(t, \pm h) = \mathbf{H}(t, \pm h), && t \in \mathcal{C}. \end{aligned} \tag{1}$$

where $\mathfrak{T}(t, \nabla)$ is the traction operator and $\mathbf{U} = (U_1, U_2, U_3)^\top$ is the displacement.

The object of the investigation is what happens with the boundary value problems (1) when the thickness of the layer diminishes to zero $h \to 0$. Is proved the following.

Theorem. *Let the weak $\mathbb{L}_2$–limits*

$$\lim_{h\to 0} \mathbf{F}(t, h\tau) = \mathbf{F}(t), \qquad \lim_{h\to 0} \frac{1}{2h} \left[\mathbf{H}(t, +h) - \mathbf{H}(t, -h)\right] = \mathbf{H}^{(1)}(t) \tag{2}$$

exist. Then the boundary value problem (1) converges in the sense of Γ-limit to the following BVP on the mid surface $\mathcal{C}$

$$\begin{cases} \mu \left[\Delta_{\mathcal{C}}\overline{U}_\alpha + \mathcal{D}_\beta\mathcal{D}_\alpha\overline{U}_\beta - 2\mathcal{H}_{\mathcal{C}}\nu_\beta\mathcal{D}_\alpha\overline{U}_\beta - \mathcal{D}_\gamma(\nu_\alpha\nu_\beta\mathcal{D}_\gamma\overline{U}_\beta)\right] \\ \quad + \dfrac{4\lambda\mu}{\lambda + 2\mu}\left[\mathcal{D}_\alpha\mathcal{D}_\beta\overline{U}_\beta - 2\mathcal{H}_{\mathcal{C}}\nu_\alpha\mathcal{D}_\beta\overline{U}_\beta\right] = \dfrac{1}{2}F_\alpha + H_\alpha^{(1)} \quad \text{on} \quad \mathcal{C}, \\ \overline{U}_\alpha(t) = 0 \qquad\qquad \text{on} \quad \Gamma = \partial\mathcal{C}, \quad \alpha = 1, 2, 3. \end{cases} \tag{3}$$

where $\overline{\mathbf{U}}(t) := (\overline{U}_1(t), \overline{U}_2(t), \overline{U}_3(t))^\top$, $\overline{U}_\alpha(t) := U_\alpha(t, 0)$, $\alpha = 1, 2, 3$ and λ, μ are the Láme constants.

The operator in the ***shell equation*** (3) *is positive definite and the boundary value problem is uniquely solvable in the classical setting* $\overline{U} \in \mathbb{L}_2(\mathcal{C})$.

The suggested approach is based on the following: a) The variational representation of the BVP (1) (minimization problem for the energy functional); b) On the representation of the Laplace and Láme operators in terms of Günter's tangential and normal (to the surface) derivatives. Namely, if $\nu(t)$, $t \in \mathcal{C}$, is the unit normal vector field on the surface, extended in the domain Ω_h, the Günter's derivatives read $\mathcal{D}_j := \partial_j - \nu_j \mathcal{D}_4, \quad j = 1, 2, 3,$ $\mathcal{D}_4 = \partial_\nu = \sum_{k=1}^{3} \nu_k \partial_k$ and the Laplace-Beltrami operator on the surface $\mathcal{C}$ is represented as follows $\Delta_{\mathcal{C}} = \mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{D}_3^2$. Moreover, the Laplace $\Delta = \partial_1^2 + \partial_2^2 + \partial_3^2$ and the Láme operator $\mathcal{L} = -\mu\,\Delta - (\lambda + \mu)\,\nabla\mathrm{div}$ in the domain are represented as follows:

$$\Delta = \sum_{j=1}^{4} \mathcal{D}_j^2 + 2\mathcal{H}_{\mathcal{C}} \mathcal{D}_4, \qquad \mathcal{L} = -\mu\,\Delta_{\Omega^h} - (\lambda + \mu)\,\nabla_{\Omega^h} \mathrm{div}_{\Omega^h}.$$

Here $\mathcal{H}_{\mathcal{C}}$ is the mean curvature of the surface $\mathcal{C}$ and

$$\nabla_{\Omega^h} \varphi := \Big\{ \mathcal{D}_1 \varphi, ..., \mathcal{D}_4 \varphi \Big\}^\top, \qquad \mathrm{div}_{\Omega^h} \mathbf{U} := \sum_{j=1}^{4} \mathcal{D}_j U_j, \quad U_4 := \langle \nu, \mathbf{U} \rangle$$

are the gradient and divergence.

The investigation is carried out in collaboration with T. Buchukuri, (Tbilisi).

Graphs with table restrictions on reachability

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We consider the graphs in which the limitations on reachability are generated by the elements of the monoid that are mapped to the edges of the graph. Constraints are formulated in terms of sequences of edges that form admissible ways on a graph. The problems of reachability of vertices and shortest ways on such graphs are solved. The solution of these problems is to build special skan-graph and transfer the problems with restrictions on reachability on the original graph to the problem without restrictions on the scan-graph.

About the designation system in the course of linear algebra

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This work is devoted to the development of the symbolic language of linear algebra, which we consider far from perfect. The main mathematical objects which we worked are matrices, vectors (elements of linear spaces) and their coordinates, basis's of linear spaces, linear operators and their matrices, transition matrices from basis to basis. All proposed designations are "technological". They are easy to type not only in Latex, but also in the editor of mathematical formulas Microsoft Equation, standard for MS Word. Our experience of working with students has shown that this system of designations is well perceived and understood by them. We used this notation system in our textbooks [1-2].

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Leaky quantum structures: spectral properties and asymptotic expansions

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The topic of the talk are Schrödinger operators with an attractive singular 'potential', supported by a manifold or a geometric complex Γ of codimension one, formally written as $-\Delta - \alpha\delta(x - \Gamma)$. We discuss the ways in which spectral properties of such systems are influenced by the geometry of the interaction support with the main attention paid to situations when the coupling constant is large or the geometric perturbation is weak, and asymptotic expansions can be derived. We also discuss effects arising from the presence of a magnetic field, in particular, the influence of an Aharonov-Bohm flux on the so-called Welsh eigenvalues.

Scissors congruence and flexible polyhedra

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Flexible polyhedra are polyhedral surfaces with rigid faces and hinges at edges that admit non-trivial deformations, that is, deformations not induced by ambient isometries of the space. In 1996 Idzhad Sabitov proved his famous theorem claiming that the volume of any flexible polyhedron in three-dimensional Euclidean space remains constant during

the flexion. In 2012 these result was generalized by the speaker to the higher-dimensional case.

An interesting problem consists in finding other metric invariants of polyhedra that remain constant during flexions. We shall show that the Dehn invariant of any flexible polyhedron also remains constant during any its flexion. This implies that every three- or four-dimensional flexible polyhedron remains scissors congruent to itself during the flexion.

The talk is based on a recent joint work with Leonid Ignashchenko.

The classification of the medial hyperidentities

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Hyperidentities are formulas of the second order logic. In contrast to the first order logic formulas, in the second order logic we put quatifiers not only on the objective variables, but also on the functional variables. In [1,2] classifications of associative hyperidentities on e-algebras and q-algebras, as well the classification of Kolmogorov's hyperidentities and hyperidentities of right (left) transitivity are presented. The obtained result is the continuation of the results provided above. In [3] it was proved that the medial system are linear on some Abelian group. This work is based on these results, and finds necessary and sufficient conditions for the hyperidentities to be held in invertible algebras. Overall, there are 18 medial hyperidentities of functional rank 2 or 3, such that these hyperidentities are pairwise non-equivalent and every medial hyperidentity is equivalent to one of that 18 hyperidentities. Classifications of medial hyperidentities in the e-algebras and q-algebras are open problems.

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Quantum vacuum: the structure of empty space-time and quintessence with gauge symmetry group $SU(2) \otimes U(1)$

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We consider the formation of structured and massless particles with spin 1 (vector boson), by using the Yang-Mills like stochastic equations system for the group symmetry $SU(2) \otimes U(1)$ without taking into account the nonlinear term characterizing self-action. We prove that, in the first phase of relaxation, as a result of multi-scale random fluctuations of quantum fields, massless particles with spin 1, further referred as *hions*, are generated in the form of statistically stable quantized structures, which are localized on 2D topological manifolds. We also study the wave state and the geometrical structure of the *hion* when as a free particle and, accordingly, while it interacts with a random environment becoming a quasi-particle with a finite lifetime. In the second phase of relaxation, the vector boson makes spontaneous transitions to other massless and mass states. The problem of entanglement of two *hions* with opposite projections of the spins $+1$ and -1 and the formation of a scalar zero-spin boson are also thoroughly studied. We analyze the properties of the scalar field (dark energy-quintessence) and show that it corresponds to the Bose-Einstein (BE) condensate. The scalar boson decay problems, as well as a number of features characterizing the stability of BE condensate, are also discussed. Then, we report on the structure of empty space-time in the context of new properties of the quantum vacuum, implying on the existence of a natural quantum computer with complicated logic, which manifests in the form of dark energy. The possibilities of space-time engineering are also discussed.

Complex degrees of one differential operator associated to the Helmholtz operator

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We study complex powers of the differential operator of the second order with complex coefficients in the principal part.

Complex powers of this operator with negative real parts are realized as anisotropic potentials with nonstandard metric. Positive powers, reverse negative as of approximative inverse operators (AEO).

Finite group actions on simple C*-algebras

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In this talk, first we give some preliminaries on operator algebras and actions of topological groups on them. Then we focus on actions of finite groups on simple C*-algebras with the Rokhlin-type properties which can be viewed as noncommutative freeness. Finally, we give recent results on the structure of crossed products by such actions and open problems.

Some bounds on the Roman k-domination number in graphs

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Let $k \geq 1$ be an integer. A Roman k-dominating function on a graph G with vertex set V is a function $f : V \to \{0, 1, 2\}$ such that every vertex $v \in V$ with $f(v) = 0$ has at least k neighbors $u_1, u_2, \cdots, u_k$ with $f(u_i) = 2$ for $i = 1, 2, \cdots, k$. The weight of a Roman k-dominating function is the value $f(V) = \sum_{v \in V} f(v)$. The minimum weight of Roman k-dominating functions on a graph G is called the Roman k-domination number, denoted by $\gamma_{kR}(G)$. In this note, we present several new bounds on the Roman k-domination number and by using these bounds we improve some results of this topic.

On stability of bases in Frechet spaces

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We show that, for each compact set K of infinite cardinality with an isolated point, the space of Whitney jets on K does not possess a polynomial basis. On the other hand, polynomials are dense in any Whitney space. Thus, there are no general results about stability of bases in Frechet spaces.

New results in multiplicative calculus

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Many authors gave definitions of a new kind of derivative, integral and multiplicative metric and thus established a new calculus, called multiplicative calculus. In this paper we give new results of the Hahn Banach theorem in the multiplicative norm spaces and we find best approximation by bounded multiplicative functionals.

Some paradoxical phenomena in theory of probability and their applications

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The mathematical models of moving particles, describing a behavior of complicated queuing systems are considered. Motion of particles depends on a distance between them. In the capacity of an efficiency index in these systems, an average waiting time of particle in the fixed point is taken. A control function is introduced, which means delay of some particles during motion. The class of systems for which introducing of delays can reduce an efficiency index is described. The optimal function minimizing an efficiency index is found. Numerical examples demonstrating results are given.

A strategy for finding integer almost periods in the theory of almost periodic functions

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In this work, we analyse the behaviour of rationally independent real numbers and construct an integer approximation strategy for almost periods in the theory of almost periodic functions.

Properties in weak absolute geometry

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Searching for results that bear some similarity to Propositions 20 and 21 of Book I of Euclid's *Elements*, M. Hajja and H. Martini arrive at the following theorem, whose validity they prove in the real Euclidean plane.

Theorem 1. *Let P be a point in the plane of a triangle ABC. Then there exists a point Q inside or on the boundary of ABC that satisfies.*

$$AQ \leq AP, \quad BQ \leq BP, \quad CQ \leq CP. \tag{1}$$

Aware of the discrepancy between the statement of the theorem, whose notions belong to Hilbert's absolute geometry (whose axioms are the plane axioms of incidence, order, and congruence of groups I, II, and III of Hilbert's Grundlagen der Geometrie), which is where one expects a proof to be carried through, and the methods of proof used, the authors ask: "Its fanciful proof, using Zorn's lemma and the Bolzano-Weierstrass theorem, raises the question whether such a heavy machinery is indeed inevitable." Moreover, since they can only prove the existence of the point Q, they also ask "whether there is a procedure (an algorithm) to construct the point Q". Solving this problem, we prove theorems mentioned below within a very weak plane absolute geometry (all of whose axioms can be deduced inside Hilbert's plane absolute geometry).

Theorem 2. *For any point P inside or on the boundary of triangle ABC, there is no point Q, different from P, such that Q and P satisfy (1).*

Theorem 3. *For every point P outside of triangle ABC there exists a point Q inside of triangle ABC, such that Q and P satisfy (eq0.1).*

In the proof of Theorem 3 we also provide an algorithm to construct such a point Q.

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The uniqueness theorems in the inverse problems for dirac operators

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We introduce new supplementary data to the set of the eigenvalues, to determine uniquely potential matrix in the inverse problem for Dirac canonical operator. Besides, we obtain others uniqueness theorems in inverse problems, which are the analogues of well-known Borg, Marchenko and MeLaughlin–Rundell theorems in inverse Sturm–Liouville problems.

Adams type result for Riesz potential and it is commutators in the generalized weighted Morrey spaces with variable exponent

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We consider generalized weighted Morrey spaces with variable exponent. In case of unbounded sets we prove the boundedness of the Riesz potential and it is commutators in such spaces. In all the cases the conditions for the boundedness are given it terms of Zygmund-type integral inequalities on general function, which do not assume any assumption on monotonicity of general function.

Finite rigid sets in curve complexes of non-orientable surfaces

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A finite rigid set in a curve complex of a surface is a subcomplex such that every locally injective simplicial map defined on this subcomplex into the curve complex is induced from an automorphism of curve complex. In this talk, we find finite rigid sets in the curve complexes of connected, non-orientable surfaces.

Invariant subspaces of the backward shift operator in spaces of entire functions

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We describe proper closed invariant subspaces of the generalized backward shift operator in a space of entire functions of exponential type. This space is topologically isomorphic by means of the Laplace transform to the strong dual of a space of all germs of analytic functions on a convex locally closed subset of the complex plane.

On the Fredholm property of a class of convolution-type operators

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The notions of the L-convolution operator and the L-Wiener-Hopf operator are introduced by replacing Fourier transform in the definition of the convolution operator by a spectral transformation of the self-adjoint Sturm-Liouville operator on the axis L. In the case of zero potential, the introdused operators concide with the convolution operator and the Wiener-Hopf integral operator, respectively. We will discuss the Fredholm properties.

Dyadic generalized functions and applications

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In the year of 2007 dyadic generalized functions were introduced by Prof. B. Golubov as continuous linear functionals on the linear space $D_d(\mathbb{R}_+)$ of infinitely differentiable functions compactly supported by the positive half-axis $\mathbb{R}_+$ together with all dyadic derivatives.

The space of rapidly decreasing in the neighborhood of infinity dyadic generalized functions $S_d(\mathbb{R}_+)$ was defined as the space of functions from ${C^{(\infty)}}_W(\mathbb{R})$ such that for all $\alpha,\ \beta \in \mathbb{Z}_+$

$$\lim_{x\to\infty} (h(x))^{-\beta}\phi^{\alpha}(x) \ = \ 0,\ h(x) = 2^{-n},\ 2^n \le x < 2^{n+1},\ n \in \mathbb{Z}.$$

However, $D_d(\mathbb{R}_+)$ and $S_d(\mathbb{R}_+)$ defined as they are above were not invariant with respect to the Walsh-Fourier transform $F[f](x) \ = \ \int_{R_+} \psi(x,y)f(y)dy$.

Thus, S. Volosivets suggested in 2009 the following definition of the space $D_d(\mathbb{R}_+)$: functions f such that

$$f(x) = constant, \quad x \in \delta$$

for some dyadic interval δ. Now, if $f \in D_d(\mathbb{R}_+)$ then $F[f]$ also belongs $D_d(\mathbb{R}_+)$.

Thereby, we consider the space $D_d(\mathbb{R}_+)$ and prove that it cannot be defined "better" in terms of the range of functions it contains to preserve the property of being invariant. We also consider the refinement equation $\phi = \sum_{k=0}^{2^n-1} C_k \phi(2x \ominus k)$, $x \in R_+$, C_k – finite set of complex numbers, and prove some facts of its solution ϕ, in particular, the uniqueness of that solution (if exists).

Exponentiated generalized exponential Gompertz distribution

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In this paper, a new five-parameter distribution is introduced, which is the generalization of Gompertz distribution, and is called the exponentiated generalized exponential Gompertz (EGEG) distribution.This distribution includes widely used exponentiated generalized Gompertz and Gompertz distributions as sub-models. The EGEG distribution can be used effectively in modeling lifetime data, survival data, and reliability problems. In this research, some properties of this new distribution, such as moments, quantile function, survival function, hazard rate function, reliability, and order statistics are studied. Also, the parameters of the new distribution are estimated using several methods and these methods are compared by using the Monte-Carlo simulation. Finally, applications of the model are demonstrated to show the usefulness of the proposed model.

Unconditionality of periodic orthonormal spline systems in L^p

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It is proved that for every admissible sequence of knots and for every natural k the corresponding periodic orthonormal spline system of order k is an unconditional basis in $L^p(T)$ for every $1 < p < \infty$.

Biorthogonal series solution for the biharmonic equation and Stokes flow problems

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A semi-analytical approach is presented and described for the solution of a class of partial differential equations that model creeping viscous incompressible flow through cavities that arise in fluid dynamics. Such flows are modeled by the biharmomic equation. The strategy leads to the development of biorthogonality conditions and an algorithm for the computation of the coefficients in the eigenfunction expansion. Properties of solutions of the biharmonic equation as well as the more general polyharmonic equation are explored. Numerical experiments will be given in order to test and confirm the validity and applicability of the proposed strategy.

On weakly prime radical

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In this work, we will deal with the weakly prime radical of submodules and its properties. We discussed the conditions under which the weakly prime radical of a module is distributive over intersection. Also, we gave a criteria in terms of envelope, to say that every weakly quasi-primary submodule is weakly prime.

Conformal mapping of the half-plane onto circular polygon

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We solve the problem of constructing conformal mapping from the half-plane onto a polygon with boundary consisting of circular arcs. We use the Schwarz differential equation for representation of the mapping. There is a classical problem of determining parameters for the equations. They are the preimages of polygon's vertices under the mapping and additional accessory parameters. To determine these parameters, we generalize Kufarev's method, based on the Loewner's differential equation.

The system of Hermite-Pade polynomials for an algebraic function and reconstruction of its values

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For a given germ f_0 of an algebraic function f of order $(m+1)$, for each natural number n we define a system of m tuples of polynomials. These tuples are numbered by the number k = 1, ..., m, and we call them k-th polynomials of Hermite-Pade m-system (of order n)". All these polynomials are found constructively, as solutions of linear homogeneous systems, and coefficients of these systems are some linear combinations of the Taylor coefficients of the original germ f_0. It turns out that the ratio of some polynomials from the k-th set assymptotically (as $n \to \infty$) reconstructs the sum of the values of the original function f on the first k sheets of the so-called Nuttall partition of Riemann surface of f into sheets.

Thus, from an initial germ f_0 of algebraic function f we reconstruct the values of f on all sheets of the Nuttall partition of its Riemann surface, except for the "last", with the help of systems of linear algebraic equations.

We note that 1-th polynomials of Hermite-Pade m-system are well-known Hermite-Pade polynomials of the second type and m-th polynomials of Hermite-Pade m-system are well-known Hermite-Pade polynomials of the first type.

Superposition of solutions of the Navier-Stokes equations

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The Navier-Stokes equations describe motion of fluid and gaseous media in presence of viscosity. These equations are non-linear, and are of interest from a purely mathematical point of view and from the point of view of applications. Study of the Navier-Stokes equations is one of directions of modern mathematical physics. Despite the great practical significance, for today there is a whole list of purely mathematical issues requires additional study.

One of such issues is the constructing of exact solutions taking into account nonlinear terms and generating of exact solutions. In this connection, a particular question arises is a superposition of solutions possible. Whether we can generate a third exact solution of the Navier-Stokes based on two known ones.

The proposed investigation gives a resolution in general to this issue. Although the Navier-Stokes equations are non-linear, a positive answer may be given. But certain additional restrictions must be met. These restrictions represent the sufficient conditions for sum and difference of two solutions to be the solution as well. These conditions are reduced to a certain second order system of linear partial differential equations with respect to six associated unknowns and can be tested in practice. Examples are considered.

A representation of the transmutation kernels for the Schrödinger operator in terms of eigenfunctions and applications

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The representations of the kernels of the transmutation (transformation) operator and of its inverse relating the one-dimensional Schrödinger operator with the second derivative are obtained in terms of the eigenfunctions of a corresponding Sturm-Liouville problem. Since both series converge slowly and in general only in a certain distributional sense we find a way to improve these expansions and make them convergent uniformly and absolutely by adding and subtracting corresponding terms. A numerical illustration of the obtained results is given.

This is a joint work with Kira V. Khmelnytskaya and Sergii M. Torba

Continuity properties for classes of pseudo-differential operators of variable orders acting on Besov spaces

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The pseudodifferential operators of variable orders acting in Besov spaces of variable smoothness are considered. The boundedness and compactness of operators under consideration are proved.

On the stability of Thomson vortex N-gon and vortex tripole/quadrupole in geostrophic models of Bessel vortices and in two-layer rotating fluid

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A two-layer quasigeostrophic model is considered. The stability analysis of the stationary rotation of a system of N identical point vortices lying uniformly on a circle of radius R in one of layers is presented. The vortices have identical intensity and length scale is $\gamma^{-1} > 0$. The problem has three parameters: N, γR and $-1 < \alpha < 1$, where $\alpha = h_2 - h_1$ is the difference between thicknesses of the top layer h_1 and the bottom layer h_2. The total layer thickness is one. The stability of the stationary rotation is interpreted as orbital stability. The instability of the stationary rotation is instability of system reduced equilibrium. The quadratic part of the Hamiltonian and eigenvalues of the linearization matrix are studied.

The parameters space $(N, \gamma R, \alpha)$ is divided on three parts: **A** is the domain of stability in an exact nonlinear setting, **B** is the linear stability domain, where the stability problem requires the nonlinear analysis, and **C** is the instability domain. The case **A** takes place for $N = 2, 3, 4$ for all possible values of parameters γR and β. In the case of $N = 5$ we have two domains: **A** and **B**. In the case $N = 6$ part **B** is curve, which divides the space of parameters $(\gamma R, \beta)$ into the domains: A and C. In the case of $N = 7$ there are all three

domains: **A**, **B**, and **C**. The instability domain **C** takes place always if $N = 2n \geq 8$. In the case of $N = 2\ell + 1 \geq 9$ there are two domains: **B** and **C**.

The stability problem for $N+1$ vortices is considered for a two-layer quasigeostrophic model and model of homogeneous fluid. In the case of two-layer fluid the quadratic part of the Hamiltonian and eigenvalues of the linearization matrix are studied for the vortex structure consisting of a central vortex of arbitrary intensity Γ and two/three identical peripheral vortexes ($N = 2, 3$). The identical vortexes, each having a unit intensity, are uniformly distributed over a circle of radius R in a single layer. The central vortex lies either in the same or in another layer. Some new results on stability $N + 1$ vortices are obtained for Kirchhoff's model.

The stability of the Thomson vortex N-gon is also studied in the case of the model of the Bessel vortices for any $N \geq 2$.

A number of statements about the stability is obtained for the systems of interacting particles with the general Hamiltonian depending only on distances between the particles.

The main results are published in the papers [1-4].

The work was supported by the Ministry of Education and Science of the Russian Federation, Southern Federal University (Projects No. 1.5169.2017/8.9).

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d-basis and complex structures in vector lattices

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The main result states that a universally complete real vector lattice admits a band preserving complex structure if and only if it contains no locally one-dimensional bands. A description of order bounded disjointness preserving complex structures on Archimedean semiprime f-algebras is also given. as well as the necessary information about d-bases in vector lattices is presented in this talk.

Multiple Steklov eigenvalues in a domain with a small hole

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Let Ω^o be a bounded open domain of $\mathbb{R}^n$. Let ν_{Ω^o} denote the outward unit normal to $\partial\Omega^o$. We assume that the Steklov problem $\Delta u = 0$ in Ω^o, $\frac{\partial u}{\partial \nu_{\Omega^o}} = \lambda u$ on $\partial\Omega^o$ has a multiple eigenvalue $\tilde{\lambda}$ of multiplicity r. Then we consider an annular domain $\Omega(\epsilon)$ obtained by removing from Ω^o a small cavity of size $\epsilon > 0$, and we show that under appropriate assumptions each elementary symmetric function of r eigenvalues of the Steklov problem $\Delta u = 0$ in $\Omega(\epsilon)$, $\frac{\partial u}{\partial \nu_{\Omega(\epsilon)}} = \lambda u$ on $\partial\Omega(\epsilon)$ which converge to $\tilde{\lambda}$ as ϵ tends to zero, equals real a analytic function defined in an open neighborhood of $(0,0)$ in $\mathbb{R}^2$ and computed at the point $(\epsilon, \delta_{2,n}\epsilon\log\epsilon)$ for $\epsilon > 0$ small enough. Here $\nu_{\Omega(\epsilon)}$ denotes the outward unit normal to $\partial\Omega(\epsilon)$, and $\delta_{2,2} \equiv 1$ and $\delta_{2,n} \equiv 0$ if $n \geq 3$. Such a result is an extension to multiple eigenvalues of a previous result obtained for simple eigenvalues in collaboration with S. Gryshchuk.

Dependence in statistics and probability

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The goal of this talk is to promote a new method for constructing normal confidence intervals for the mean and testing hypotheses, when the data is coming from stochastic structures with possibly long memory. We will emphasize the importance of this new result for cases when the dependence structure is not known and there is no information about the density function of the family from which the observations are selected. We will show this on applications including linear processes, linear regression, reversible Markov chains with long memory, testing and estimation of the means. Each of the examples will show the risks of not considering the dependence.

Necessary conditions for an extremum in non-convex problems of the calculus of variations

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In this thesis, we propose two approaches that allow us to investigate a strong extremum in non-convex vector problems: in the simplest problem of variational calculus and in the variational problem with higher derivatives.The essence of the proposed approaches consists in the introduction of various special variations: Weierstrass variations in a modified form; variations expressed by Legendre polynomials. The necessary minimum conditions are obtained which contain the Weierstrass conditions and its localization modifications as well as Legendre conditions. In the case when the Legendre condition degenerates, the equality and inequality type of necessary conditions are obtained for weak local minimum.

Some mapping properties of multilinear integral operators

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We will be focused on the following topics of multilinear theory of integral operators: complete characterization of the boundedness of multilinear fractional integral operators defined with respect to a measure; two-weight estimates for multi(sub)linear strong maximal operators; the multilinear variants of the quantities which measure the noncompactness of multilinear operators taking values in Banach spaces with the uniform approximation property; applications to multilinear variant of the Hilbert and Riesz transform on rearrangement invariant spaces.

Some part of the research was supported by the Shota Rustaveli National Science Foundation grant (No. FR-18-2499).

RBF-FD method in a weak formulation

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Radial basis functions (RBFs) are known as a promising tool in approximation theory for reconstructing functions from scattered values. In this talk, the idea of direct discretization via RBFs is applied on a local Petrov-Galerkin test space of a partial di erential equation (PDE). This results to a weak-based RBF-generated nite di erence (RBF-FD) scheme that possesses some useful properties. The error and stability issues are considered. When the PDE solution or the basis function has low smoothness, the new method has superior accuracy over the already well established strong-based collocation methods. Although the method uses a Galerkin formulation, it still remains meshless because not only the approximation process relies on scattered point layouts but also integrations are done over non-connected, independent and well-shaped subdomains. Some applications to potential and elasticity problems on scattered data points support the theoretical analysis and show the e ciency of the proposed method.

Convex optimization over the weakly efficient set using a generalized cutting plane method

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In this talk, we study the optimization problem (PWE) of minimizing a convex function over the set of weakly efficient solutions of a convex multiobjective problem. We do this by using the fact that each lower semicontinuous convex function is an upper envelope of its affine minorants together with a generalized cutting plane method. We give necessary conditions for optimal solutions of the problem (PWE). Moreover, for the first time by using a generalized cutting plane method an algorithm for solving the problem (PWE) and numerical results are presented. We also prove that the proposed algorithm terminates after a finite numbers of iterations, and the algorithm is coded in MATLAB language and evaluated by some numerical examples.

Orthogonally additive 2-homogeneous polynomials on some space

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We investigate orthogonally additive 2-homogeneous polynomials on a generalized matrix algebra. We then examine our results for a triangular algebra and also for the full matrix algebras.

Codimension-2 bifurcations in the dynamic systems with circular symmetry

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The investigation of codimension-2 bifurcations in the dynamic systems with symmetry enables us to find out new types of secondary regimes branching off from the symmetric regimes. The central manifold approach combined with the reduction to the normal form lead to the so called amplitude systems. These ODE systems describe the nonlinear interaction between the neutral modes and include always several nonlinear terms due to so called intrinsic resonances. However, sometimes additional resonances appear. We present the results of investigation of interaction between two oscillation neutral modes in the systems with circular symmetry.

Boolean and De Morgan structures, their applications

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Theorem 1. (R. Dedekind, 1897). *The free Boolean algebra on n free generators is isomorphic to the Boolean algebra of Boolean functions of n variables.*

Theorem 2. (R. Dedekind, 1897). *The free bounded distributive lattice on n free generators is isomorphic to the bounded lattice of monotone Boolean functions of n variables.*

A problem posed by B.I. Plotkin in 1970s has required finding the varieties of algebras with analogous functional representations of free finitely generated algebras. In this talk we give a solution of this problem.

3D mathematical model of the deep natural stream as a sequence of 2D special BVP for Laplace and Poisson equations

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A reduced 3D mathematical model of a natural stationary channel stream is considered.

It is believed that the channel bed is defined by a weakly varying and smooth function, and the flow itself is long and deep, i.e. the length of the channel section in question is much larger than its width and depth, which, in turn, have similar dimensions.

Using the previously developed technique for obtaining reduced mathematical models for channel flows, it is possible, at constant viscosity, to reduce the solution of a complex 3D problem to the solution of a sequence of standard 2D boundary-value problems for Laplace and Poisson equations on a cross-section.

This work was supported by the Vladimir Potanin Foundation, project ID GK190000844. The results of the study will be used in the Master Program "Mathematical Modeling, Numerical Methods and Program Complexes".

From undergraduate to postgraduate school: the problem of selection and recruitment of applicants

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The results of recent years of admission companies at the Institute of Mathematics, Mechanics and Computer Science of the Southern Federal University indicate a loss of interest among applicants in such fundamental areas of study as mathematics and mechanics. Negative trends relate to all levels of training, from undergraduate to postgraduate school.

The modern three-stage system of HE, when students are taught in undergraduate studies for 4 years, master school for 2 years and postgraduate school for 4 years, requires coordination of educational trajectories between different levels. According to the author, not always a magistracy can be considered as the final preparation for professional activity, when the studied disciplines provide narrow specialization in the chosen professional field.

The discussion focuses on the relationship of master's level and postgraduate studies for the training of specialists focused on research activities, defending a dissertation and a scientific and pedagogical career in high-tech industries, research organizations and universities.

It is proposed to consider master's level studies as a preparatory stage for further studies in postgraduate school. In this case, it is justified to strengthen the fundamental training and organization of the project activities of undergraduates with the involvement of postgraduate students. The combination of the project-oriented and fundamental content of training is the essence of the concept of a variable-modular construction of the OOP of scientific master's programs.

Reduce the semi-Markovian random walk processes

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In this abstract, we reduce the semi-Markovian random walk processes with jumps and delaying screen in zero. The Laplace transform on time, Laplace-Stieltjes transform on phase of the conditional distribution of semi-Markovian random walk processes with jumps is found. We get a mathematical modeling of the semi-Markov random walk processes with a delaying screen in zero, given in general form by means of integral equation. The purpose of this abstract is to reduce the fractional order integral equation to a fractional order differential equation. Finally, we find the exact solution of fractional order differential equation.

Legendrian knots in contact 3-manifolds

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Contact geometry has long been an essential tool in the study of the topology of low-dimensional manifolds. A contact structure on an odd-dimensional manifold is a maximally non-integrable, non-vanishing hyperplane field. When the manifold has dimension three, there is a special class of knots which are everywhere tangent to the contact planes.

These knots are called Legendrian knots and they reveal the geometry and topology of the underlying contact 3-manifold. In this talk, I will give a brief introduction on contact 3-manifolds and Legendrian knots in contact 3-manifolds. I will discuss new invariants for Legendrian knots as well as new classification results for Legendrian knots.

Discrete homotopy analysis method for fractional difference equations

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The discrete homotopy analysis method (DHAM) is proposed to solve linear and nonlinear fractional partial difference equations. Fractional differences are described by Caputo's sense. Test examples demonstrate the applicability of proposed method. Presented results show that this method is efficient and powerful in solving wide classes of fractional difference equations.

Initial-boundary value problems for the biharmonic nonlinear Schrödinger equation

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We discuss the local and global wellposedness of initial boundary value problems for the biharmonic nonlinear Schrödinger equation on the half line and finite interval. A representation formula for the linear problem on the half line is found by Fokas's unified transform method, while the finite interval problem is treated by the Laplace transform in time. We prove space and time trace estimates in fractional Sobolev spaces. Strichartz estimates are obtained to deal with the low regularity case. The classical contraction argument yields local wellposedness for corresponding nonlinear models with power type nonlinearity. Some global wellposedness theorems are also given for a restricted set of power indices.

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Boundedness of the harmonic Bergman projection in various Banach spaces with non-standard function spaces

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We introduce and give the definitions and notations of variable exponent Lebesgue spaces, Orlicz spaces, Grand and Small Lebesgue space of harmonic functions. We give the proof of the boundedness of Bergman projection in various Banach spaces of functions on the unit disc in the complex plain, in the case of variable exponent Lebesgue space, Orlicz space and generalized Grand spaces is proved and estimates of the growth of these functions.

Abelian Higgs models on compact Riemann surfaces

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Abelian Higgs models on compact Riemann surfaces are generalization of the (2+1)-dimensional Abelian Higgs model on the plane, which arises in the theory of superconductivity. The static solutions of these models are minima of the potential energy. If the Chern number of the underlying line bundle of the model is a natural number N, then up to gauge equivalence such solutions are given by N points of the Riemann surface (not necessarily distinct). So the moduli space of static solutions is an N-dimensional complex manifold (namely, the N-th symmetric power of the Riemann surface).

The kinetic energy functional of the model defines some specific Riemann metric of the moduli space of the static solutions (the kinetic metric). According to so-called adiabatic principle, the geodesics of the kinetic metric give rise to approximations for "slow" dynamical solutions of the model. This allows one to study the dynamics in the model with the help of the properties of the metric. Unfortunately, the adiabatic principle for models on compact Riemann surfaces has not been yet strictly justified in general.

Asymptotic analysis of the six-dimensional model of free convection in a spheroidal cavity with vanishing dissipation

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A six-dimensional model of free convection of a viscous heat-conducting fluid in an ellipsoidal cavity is considered. In the case of an inviscid, non-heat-conducting fluid, this model reduces to the well-known Euler-Poisson equations of motion of a rigid body with a fixed point. At the same time, unlike other low-mode models, it determines the exact solutions of the free convection equations in the Oberbeck-Boussinesq approximation with spatially linear velocity and temperature fields. The ellipsoidal cavity is assumed to be rotationally symmetric, the temperature gradient at its boundary and the force of gravity are directed along the axis of symmetry. If the coefficients of viscosity and thermal conductivity are small, then the model can be considered as a perturbation of the Euler-Poisson equations in the integrable Lagrange case. This allows us to perform an

asymptotic analysis for it in the limiting case of vanishing dissipation. It turns out that, depending on the parameters of the problem and the initial conditions, either stationary or periodic motions are established. The first ones are "elliptical" rotations of fluid around an axis orthogonal to the direction of gravity. The second ones are oscillatory-type motions with zero mean angular velocity. For them, the axis of rotation is constant (orthogonal to gravity), and the intensity and direction of rotation change periodically.

Discriminants and polytopes in toric geometry

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Toric varieties play an important role at the crossroad of algebra, geometry and combinatorics. Toric geometry is both rigid and rich, and allows for testing conjectures and proving results in algebraic geometry using combinatorial methods, and in combinatorics using algebro-geometrical methods. An example of the latter is a combinatorial characterization of certain Cayley polytopes.

This talk will survey recent work by various authors concerning the description and characterization of dual and higher order dual varieties of projective toric varieties in terms of the corresponding lattice configurations and polytopes.

Convergence acceleration of trigonometric expansions: quasi-periodic interpolations and approximations

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We consider the problem of function reconstruction using its finite number of Fourier coefficients or discrete Fourier coefficients. It is well known that the convergence of truncated Fourier series or trigonometric interpolation is very slow due to the Gibbs phenomenon if the function is smooth but non-periodic on a finite interval [-1,1].

Different approaches are known for convergence acceleration of trigonometric expansions. We discuss a method leading to quasi-periodic interpolations and approximations, where the function is extended outside of [-1,1] to an interval [-a,a], a¿1 with further application of trigonometric functions with bigger periodicity 2a¿2, but tending to 2 as the number of the expansion terms tend to infinity. Unknown parameters in the expansions could be determined by solution of a system of linear equations. This approach was accomplished successfully in a series of papers for trigonometric interpolation (see [1]-[6]).

In case of Fourier approximation, different quadrature rules are considered to calculate the Fourier coefficients on [-a,a]. In case of simple quadrature rule with a uniform grid, the resultant system of linear equations has Vandermonde matrix leading to an explicit solution.

For both methods, we show exact estimates for the asymptotic errors for comparison with the classical expansions. We discuss also the results of numerical experiments.

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Composition operators on holomorphic spaces of Hölder type functions

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We study the boundedness and compactness of composition operators in the generalized Hölder type space of holomorphic functions in the unit disc with prescribed modulus of continuity and in the variable exponent generalized Hölder spaces of holomorphic functions in the unit disc.

Geometry and topology of the structure and maturation of viral capsids with icosahedral symmetry

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Viral capsid [1] consisting of one or more protein shells is an important part of the virus that protect and transport its genome to a victim cell. The proteins packaging in capsids of many viral families can be considered as the transfer of the hexagonal plane order onto the icosahedron surface.

The report discusses the in vivo and in vitro stages of formation (maturation) of a viral capsid on the example of bacteriophage HK97 [2]. An electromechanical model that describes changes in the morphology of the viral capsid during maturation and with changes in the pH of the environment is proposed. Minimization of the sum of elastic and electrostatic free energies leads to equilibrium shapes of viral shells that depend on a single elastic parameter and the detailed configuration of the imbedded protein charges. Based on the in vitro shell reconstructions of bacteriophage HK97 the details of the reversible transition between Prohead II and Expansion Intermediate II states of the HK97 procapsid induced by pH changes are considered.

The report also considered structural peculiarities and hidden symmetries [3] of multilayered viral capsids using the example of anomalous bluetongue virus capsid. Each of its three concentric shells violates the paradigmatic geometrical model of Caspar and Klug that is otherwise well suited to describe most of the known viral shells. The commensurability and structural stability of these three icosahedral spherical lattices are analyzed. It is shown that the proximity of the outer shell to its destabilization assists in the detachment of VP2 trimers from the surface of bluetongue virus capsid during the infection of the host cell. An electrostatic mechanism that can aid in this detachment is discussed.

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Around harmonic spheres conjecture

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Harmonic spheres conjecture establishes a correspondence between Yang–Mills G-fields on $\mathbb{R}^4$ and harmonic maps of the Riemann sphere S^2 into the loop space ΩG of the group G. It is an extension to general Yang–Mills G-fields of the Atiyah–Donaldson theorem establishing a correspondence between the moduli space of G-instantons on $\mathbb{R}^4$ and holomorphic maps $S^2 \to \Omega G$.

In our talk we present an approach to the proof of this conjecture based on the adiabatic limit construction using a nice parametrization of the sphere $S^4 \setminus S^1$ with one deleted circle proposed by Jarvis and Norbury. With the help of this construction one

can associate in a natural way with arbitrary Yang–Mills G-field on S^4 a harmonic map of the sphere S^2 to the loop space ΩG.

Sum and intersection some ideal Frechet spaces

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The article studies two operations (sum and intersection) defined on the category of pairs of spatial ideals. Studied their properties. The results obtained are used to study the properties of invariant classes (DN), $\underline{DN}$, $(\overline{\Omega})$, (Ω).

On the stability of the second order stochastic differential equations

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We consider the question of stability of second order stochastic differential equations. For such equations we derive sufficient conditions of stability in probability and exponential stability in mean square. As example a linear oscillator in which one of its parameters is perturbed by white noise is considered. For this one necessary and sufficient conditions of exponential stability in mean square are expressed in terms of the oscillator's parameters.

On the coincidence of the spectra of the exponents of oscillations of conjugate differential systems

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In 2009 I. N. Sergeev introduced exponents of oscillations: the full and vector frequencies of the vector function $\mathbf{x}$. Counting the full and vector frequencies of the vector function occurs by averaging the number of zeros of the projection of this function on any line. In this case the line is chosen so that the average value obtained is minimal. If the mentioned minimization is performed before averaging, so the vector frequency is derived, and if it is performed after one, then we have the full frequency. In their geometrical meaning, these characteristics of oscillation are responsible for the frequency of rotation of the vector $\mathbf{x}$ around zero.

It is known that the spectra of Lyapunov exponents of mutually conjugate regular differential systems are symmetric with respect to zero. Taking into account the non-negativity of the exponents of oscillation on the set of all solutions of differential systems, it would be interesting to answer the question: is it possible knowing the spectrum of a exponents of oscillation of a certain linear differential system, reconstruct the spectrum of this exponents of oscillation of a conjugate system? It turned out that the spectra of each exponent of oscillation of mutually conjugate systems of differential equations with continuous bounded coefficients on the positive semi-axis coincide with each other.

Strong convergence and summability of Walsh-Fourier series in martingale hardy spaces

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Unlike classical theory of Fourier series which deals with decomposition of a function into continuous waves the Walsh functions are rectangular waves. Such waves have already been used frequently in the theory of signal transmission, multiplexing, filtering, image enhancement, codic theory, digital signal processing and pattern recognition. The development of the theory of Vilenkin-Fourier series has been strongly influenced by the classical theory of trigonometric series. Because of this it is inevitable to compare results of Vilenkin series to those on trigonometric series. There are many similarities between these theories, but there exist differences also. Much of these can be explained by modern abstract harmonic analysis, which studies orthonormal systems from the point of view of the structure of a topological group.

This lecture is devoted to derive necessary and sufficient conditions for the modulus of continuity such that partial sums and some classical summability methods with respect to one- and two-dimensional Walsh-Fourier series converge in norm. Moreover, we also present some strong convergence theorems of partial sums and some classical summability methods of the one- and two-dimensional Walsh system.

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Modulus of continuity and boundedness of sub-sequences of Vilenkin-Fejer means in the martingale Hardy spaces

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The talk is devoted necessary and sufficient condition for the modulus of continuity for which subsequences of Fejer means with respect to Vilenkin systems are bounded from the Hardy space H_p to the Lebesgue space L_p, for all $0 < p < 1/2$.

An upper estimate for the chromatic number of the two-dimensional sphere

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The chromatic number of the sphere of a given radii is the minimum number of colors needed to color all points of the sphere so any two points with distance 1 apart are colored differently.

Obviously, it depends of a radii. For instance, if a radii is less than 1/2, one color is sufficient, but if a radii is large enough, then the sphere admits an embedding of the Moser's spindle, and the chromatic number is greater than three.

In this talk we consider a kind of explicit colorings of the sphere in 8 or 9 colors satisfying the condition above. This colorings are based on the solution database for the Thomson's problem.

The distribution of primes

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The main results on the distribution of primes from antiquity up till now will be presented. The talk is directed to a general mathematics audience. The focus will be on small gaps between prime numbers. The earlier major milestones in the topic are the prime number theorem, the Hardy-Littlewood prime tuples conjecture and the Bombieri-Vinogradov theorem. In the beginning decades of the 21st century some important developments on the proof of existence of small gaps between consecutive primes took place. Now it has been established that there exist infinitely many bounded gaps (of size not exceeding 246) and there is still some possibility of improvement.

An effective metaheuristic for solving heterogeneous fixed fleet open vehicle routing problem

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The heterogeneous fixed fleet open vehicle routing problem (HFFOVRP) is a well-known extension of the open vehicle routing problem (OVRP) which nowadays has drawn enormous interest from researchers because of its vital role in planning of distribution systems and logistics. In this problem, the objective is to minimize cost routes for a fleet of the various types of vehicles. In this paper, a new mixed integer programming model of the HFFOVRP and due to its NP-hard nature, a modified ant colony optimization (MACO) are proposed. In contrast to the classical ant colony optimization (ACO), the MACO uses a new state transition rule, a new ranking method for ants and a modified global updating, which is equipped with diversification and intensification mechanisms. Furthermore, the MACO was mixed with insert, swap and 2-opt algorithms. An extensive numerical experiment is performed on 38 benchmark problem instances involving up to 100 customers. The results indicate that the algorithm is successful in finding solutions within almost 0.21% of the best known solutions. Additionally, the proposed HACO finds nine new best solutions and is competitive with six exact, heuristic and metaheuristic algorithms for solving the HFFOVRP.

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