International Conference

# HARMONIC ANALYSIS AND APPROXIMATIONS, VII

## Dedicated to 90th Anniversary of ALEXANDR TALALYAN

16 - 22 September, 2018

Tsaghkadzor, Armenia

# A B S T R A C T S

Yerevan, 2018

International Conference

# HARMONIC ANALYSIS AND APPROXIMATIONS, VII

## Dedicated to 90th Anniversary of ALEXANDR TALALYAN

16 - 22 September, 2018

Tsaghkadzor, Armenia

Yerevan, 2018

*Organizers:*

**Host Institutions**

Yerevan State University
Institute of Mathematics NAS RA

**Scientific Committee**

Norair Arakelian (Armenia), Paul Gauthier (Canada), Gegham Gevorkyan (Armenia), Boris Kashin (Russia), Vakhtang Kokilashvili (Georgia), Sergei Konyagin (Russia), Michael Lacey (USA), Wolfgang Luh (Germany), Alexander M. Olevskii (Israel), Valentin Skvortsov (Russia), Vladimir Temlyakov (USA), Vilmos Totik (USA), Przemysław Wojtaszczyk (Poland)

**Organizing Committee**

Artur Sahakian (Armenia), Rafayel Barkhudaryan (Armenia), Aram Hakobyan (Armenia), Michael Poghosyan (Armenia)

*Sponsors:*

- CDC/IMU
- The Research Mathematics Fund
- Promethey Science Foundation, Armenia
- State Committee of Science MES RA
- "Hyur Service" LLC

ALEXANDR TALALYAN

1928 – 2016

# CONTENTS

# On the Riemann Boundary Value Problem in the Half-Plane for Weighted Spaces

S. Aghekyan and H. Hayrapetyan
(Yerevan State University, Armenia)
smbat.aghekyan@gmail.com, hhayrapet@gmail.com

We consider the Riemann boundary value problem in the upper half-plane of the complex plane z for weighted spaces $C(\rho)$, where $\rho(x) = \prod_{k=1}^{m} \left| \frac{x - x_k}{x + i} \right|^{\alpha_k}$, and $\alpha_k$ and $x_k$, $k = 1, 2, ..., m$ are real numbers. The problem is to find a function $\Phi(z)$ analytic in the upper and lower half-planes satisfying the condition

$$\lim_{y \to +0} \|\Phi^+(x + iy) - a(x)\Phi^-(x - iy) - f(x)\|_{C(\rho)} = 0,$$

where $f \in C(\rho)$, and $a(x) \in C^\alpha(-\infty; +\infty)$ is a function satisfying $a(x) \neq 0$, the limit $\lim_{|x| \to \infty} a(x) = a(\infty)$ exists, and $|a(x) - a(\infty)| < C|x|^{-\delta}$, for $|x| > A > 0$.

The normal solvability of this problem is established.

# References

[1] **Kazarian K.S.**, Weighted norm inequalities for some classes of singular integrals., *Studia Math.* Vol-86, N.3 (1987), 311-317.

[2] **Aghekyan S.A**, On a Hilbert problem in the half-plane in the class of continuous functions, *Proceedings of the Yerevan State University, Physical and Mathematical Sciences* N.2 (2016), 9-14.

[3] **Hayrapetian H.M.**, Dirichlet problem in the half-plane for RO-varying weight functions. Topics in Analysis and its Aplications., *Dordrecht /Boston/ London. Kluwer Academic Publishers.* Vol-147 (2004), 311-317.

# Calculation of Geometric Probabilities for Balls In $\mathbf{R}^n$

N. Aharonyan
(Yerevan State University, Armenia)
narine78@ysu.am

Let $\mathbf{R}^n$ $(n \geq 2)$ be the $n$-dimensional Euclidean space, $\mathbf{D} \subset \mathbf{R}^n$ be a bounded convex body with inner points, and $V_n$ be the n-dimensional Lebesgue measure in $\mathbf{R}^n$.

**Definition 1.** *The function*

$$C(\mathbf{D}, h) = V_n(\mathbf{D} \cap (\mathbf{D} + h)), \quad h \in \mathbf{R}^n,$$

*is called the covariogram of the body* $\mathbf{D}$. *Here* $\mathbf{D} + h = \{x + h, \, x \in \mathbf{D}\}$.

Let $S^{n-1}$ denote the $(n-1)$-dimensional sphere of radius 1 centered at the origin in $\mathbf{R}^n$. Let $\Pi r_{\mathbf{u}^\perp} \mathbf{D}$ be the orthogonal projection of $\mathbf{D}$ onto the hyperplane $\mathbf{u}^\perp$ (here $\mathbf{u}^\perp$ stands for the hyperplane with normal $\mathbf{u}$, passing through the origin). Denote by $\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D})$ the probability that the random segment $L(\mathbf{u}, \omega)$ (of fixed length $l$ and direction $\mathbf{u}$) entirely lies in the body $\mathbf{D}$.

**Proposition.** *(see [1]). Probability* $\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D})$ *in the terms of the covariogram of body* $\mathbf{D}$ *has the form:*

$$\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D}) = \frac{C(\mathbf{D}, \mathbf{u}, l)}{V_n(\mathbf{D}) + l \, b_{\mathbf{D}}(\mathbf{u})},$$

where $b_{\mathbf{D}}(\mathbf{u}) = V_{n-1}(\Pi r_{\mathbf{u}^\perp} \mathbf{D})$.

Using the explicit form of covariogram for $\mathbf{B_n}(\mathbf{R})$ we obtain

$$\mathbf{P}(L(\omega) \subset \mathbf{B_n}(\mathbf{R})) = \frac{2R}{\left(R \frac{\sqrt{\pi}\Gamma((n+1)/2)}{\Gamma(n/2+1)} + l\right)} \int_0^{\phi} \sin^n \theta \, d\theta,$$

where $\phi = \arccos \frac{l}{2R}$ and $\Gamma(x)$ is the Gamma function.

# References

[1] N. G. Aharonyan, V. K. Ohanyan, Calculation of geometric probabilities using Covariogram of convex bodies. *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 53 (2), pp. 112–120, 2018.

## Optimal uniform approximation on the unbounded domains by harmonic functions

S. Aleksanyan
(Institute of Mathematics, NAS of Armenia)
asargis@instmath.sci.am

In this talk we discuss the problem of the optimal uniform approximation on the unbounded domains $E \subset \mathbb{R}^2$ by harmonic functions. The approximable function is harmonic on the interior of $E$ and has some smoothness on the boundary of $E$. The estimations of the growth of the approximating harmonic functions on $\mathbb{R}^2$ depend on the growth of the approximable function on $E$ and it's smoothness on the boundary of $E$.

In the cases $E = S_h$ (a stripe of the width $2h$) and $E = \Delta_\alpha$ (the sector with the angle $\alpha$) we construct harmonic functions defined on $\mathbb{R}^2$ which uniformly approximate the given function on $E$ in the optimal way.

The problem of uniform approximation on the sector by the entire functions was investigated by H. Kober [1], M.V. Keldysh [2], Mergelyan [3], N. Arakelian [4] and other authors, and in the case on the stripe it was investigated by N. Arakelian [5]. The analogous problem in the case for the meromorphic functions was discussed in [6] (on the stripe) and [7] (on the sector).

# References

[1] H. Kober, *Approximation by integral functions in the complex plane,* Trans. Amer. Math. Soc., vol. 54,(1944), 7-31.

[2] M. V. Keldysh, *On approximation of holomorphic functions by entire functions* (Russian). Dokl. Akad. Nauk SSSR, **47** no. 4 (1945), 239-241.

[3] S. N. Mergelyan, *Uniform approximations to functions of a complex variable* (Russian). Uspekhi Mat. Nauk**7**, no. 2(48) (1952), 31-122; English transl in Amer. Math. Soc. Transl. (1) **3** (1962), 294-391.

[4] N. U. Arakelian, *Uniform approximation by entire functions with estimates of their growth* (Russian), Sibirski Math. Journ., vol. 4, no.5 (1963), 977-999.

[5] N. Arakelian and H. Shahgholian, *Uniform and tangential approximation on a stripe by entire functions, having optimal growth.* Computational Methods and Function theory, vol. 3, No 1 (2003), 359-381.

[6] S. H. Aleksanian, *Uniform and tangential approximation on a stripe by meromorphic functions, having optimal growth.* Journal of Contemporary Mathematical Analysis NAS of RA, 2011, vol. 46, No 2, 319-328.

[7] S. Aleksanyan, *Uniform and tangential approximation on an angle by meromorphic functions, having optimal growth*, Journal of Contemporary Mathematical Analysis NAS of RA, 2014, vol. 49, No 4, pp 3-16.

## Bounds For Marcinkiewicz Integral Operators

A. Al-Salman
(Sultan Qaboos University, Oman)
alsalman@squ.edu.om

Marcinkiewicz integral operators are of very special interest in harmonic analysis. The early study of their $L^p$ mapping properties was mainly limited to the one dimensional setting. The work of Littlewood, Paley, Zygmund, and Marcinkiewicz between 1930 and 1939 represents the basic theory of these operators. In 1958, E.M. Stein introduced Marcinkiewicz integrals in higher dimensions. Since then, the theory of Marcinkiewicz integrals has attracted the attention of many mathematicians.

Let $\mathbb{R}^n$, $n \geq 2$ be the $n$-dimensional Euclidean space and $\mathbb{S}^{n-1}$ be the unit sphere in $\mathbb{R}^n$ equipped with the induced Lebesgue measure $d\sigma$. Let $\Omega$ be a homogeneous function of degree zero on $\mathbb{R}^n$ that is integrable over $\mathbb{S}^{n-1}$ and satisfies $\int_{\mathbb{S}^{n-1}} \Omega(y')d\sigma(y') = 0$ where $y' = |y|^{-1}y$ for $y \neq 0$. The classical Marcinkiewicz integral operator introduced by E. M. Stein is given by

$$\mu_\Omega f(x) = \left( \int_{-\infty}^{\infty} \left| \int_{|y|\leq 2^t} f(x-y) \, |y|^{1-n} \, \Omega(y') dy \right|^2 2^{-2t} dt \right)^{\frac{1}{2}}. \qquad (1)$$

E. M. Stein proved that $\mu_\Omega$ is bounded on $L^p(\mathbb{R}^n)$ *for* $1 < p \leq 2$ provided that $\Omega \in Lip_\alpha(S^{n-1})$, $(0 < \alpha \leq 1)$. Subsequently, A. Benedek, A. Calderón, and R. Panzone proved the $L^p$ boundedness of $\mu_\Omega$ for all $1 < p < \infty$ provided that $\Omega$ is continuously differentiable on $\mathbb{S}^{n-1}$. In 1972, T. Walsh proved that if $\Omega \in L(\log^+ L)^{\frac{1}{2}}(\mathbb{S}^{n-1})$, then $\mu_\Omega$ is bounded on $L^2(\mathbb{R}^n)$. Furthermore, Walsh showed that the condition

$$\Omega \in L(\log^+ L)^{\frac{1}{2}}(\mathbb{S}^{n-1})$$

is optimal in the sense that the $L^2$ boundedness of $\mu_\Omega$ may fail if the condition $\Omega \in L(\log^+ L)^{\frac{1}{2}}(\mathbb{S}^{n-1})$ is replaced by $\Omega \in L(\log L)^{\frac{1}{2}-\varepsilon}(\mathbb{S}^{n-1})$ for some $\varepsilon > 0$. In 2002, Al-Salman, Al-Qassem, Cheng, and Pan showed that the condition $\Omega \in L(\log^+ L)^{\frac{1}{2}}(\mathbb{S}^{n-1})$ is also sufficient for the $L^p$ boundedness of $\mu_\Omega$ for all $p \in (1, \infty)$.

In this talk we are interested in Marcinkiewicz integral operators that are defined by translates of certain subvarieties. More precisely, for a suitable mapping $\Phi : \mathbb{R}^n \to \mathbb{R}^d$, $d \geq 1$, consider the Marcinkiewicz integral operator

$$\mu_{\Phi,\Omega} f(x) = ( \int_{-\infty}^{\infty} \left| \int_{|y|\leq 2^t} f(x-\Phi(y)) \, |y|^{-n+1} \, \Omega(y) dy \right|^2 2^{-2t} dt )^{\frac{1}{2}}. \qquad (2)$$

The main problem concerning the operators in (2) is that under what conditions on the functions $\Phi$ and $\Omega$, the corresponding operator $\mu_{\Phi,\Omega}$ maps $L^p$ into $L^p$ for some $1 < p < \infty$? In our talk we shall present several answers to this problem. Moreover, we shall highlight some open questions.

# On Construction of Periodic Wavelet Frames*

P. Andrianov and M. Skopina
(Saint Petersburg State University, Russia)
p.andrianov@spbu.ru, skopinama@gmail.com

In the present paper we consider periodic wavelet systems. A natural way to define such systems is to periodize standard wavelets, and such periodic wavelets are widely studied. However, many periodic objects of wavelet nature can not be obtained this way, and because of that, a lot of approaches to defining a periodic multiresolution analysis in a more general sense were developed. Different definitions of a PMRA and related periodic wavelet systems were introduced and studied in the literature. We consider the most general definition, which was given by M. Skopina in [1]. In the framework of this definition we provide sufficient conditions under which provided wavelet systems form dual frames (see [2]). Basing on this result, we provide a way of constructing periodic dual wavelet frames, starting from a suitable sequence of Fourier coefficients. It takes a form of the following theorem, whose proof consists of an algorithm for building such systems.

**Theorem 1.** *Let $\varphi_1$ be a 1-periodic function with the Fourier coefficients given by*

$$\widehat{\varphi_1}(l) = \begin{cases} a_0, & \text{if} \quad l = 0, \\ a_l\left(\frac{1}{|l|}\right)^{\alpha}, & \text{if} \quad l \in Q, \\ 0, & \text{otherwise}, \end{cases}$$

*where $\alpha > 1/2$, $0 < C_1 \leqslant |a_l| \leqslant C_2$ for $l = 0$ and all $l \in Q$, $Q$ is a subset of odd integers containing $1$ and satisfying the condition:*

***(Z)*** If $l \notin Q$ and $l \in I_j$ for some $j \in \mathbb{N}$, then $l + 2^j k \notin Q$ for every $k \in \mathbb{Z}$.

*Then there exist scaling sequences $\{\varphi_j\}_{j=0}^{\infty}$, $\{\widetilde{\varphi}_j\}_{j=0}^{\infty}$ generating wavelet systems $\{\varphi_0\} \cup \{\psi_{jk}\}_{j,k}$ and $\{\widetilde{\varphi}_0\} \cup \{\widetilde{\psi}_{jk}\}_{j,k}$ which are dual frames.*

*This research is supported by Volkswagen Foundation

# References

[1] M. Skopina, Multiresolution analysis of periodic functions. East Journal On Approximations, Volume 3, Number 2 (1997), 203-224.

[2] P. Andrianov, On sufficient frame conditions for periodic wavelet systems, International Journal of Wavelets, Multiresolution and Information Processing, 16, 1850002 (2018).

## The Sine Transform in Convex Geometry

R. Aramyan
(Russian-Armenian University, Armenia)
rafikaramyan@yahoo.com

The cosine transform naturally arises in mathematics. The cosine representation of the support function of a convex body plays a fundamental role in integral geometry and a number of related areas (see e.g., [1], [2]). We study (in a sense dual) the sine representation for the support function of a centrally symmetric convex body. Also an inversion formula for the sine transform is found.
We denote by $\mathbf{R}^n$ $(n \geq 2)$ the Euclidean $n$ - dimensional space. Let $\mathbf{S}^{n-1}$ be the unit sphere in $\mathbf{R}^n$.
The most useful analytic description of a convex body is the support function, because it is well known (see [1]) that a convex body $\mathbf{B}$ is uniquely determined by its support function. We prove the following theorem.

**Theorem 1.** *The support function $H(\cdot)$ of a sufficiently smooth origin symmetric convex body $\mathbf{B}$ has the following representation*

$$H(\xi) = \int_{\mathbf{S}^{n-1}} \sin(\widehat{\xi,\Omega})\, h(\Omega)\, \lambda_{n-1}(d\Omega), \quad \xi \in \mathbf{S}^{n-1} \tag{1}$$

*with an even continuous function $h(\cdot)$ (not necessarily positive) defined on $\mathbf{S}^{n-1}$.*

Note that $h$ is unique. Here by $(\widehat{\xi,\Omega})$ we denote the angle between two directions.

The right-hand side of (1) is called the sine transform of $h$ and denoted by $(Qh)(\cdot)$. The transform

$$Q : \mathcal{C}_c^\infty \longrightarrow \mathcal{C}_c^\infty$$

is injective. $h$ (in (1)) is called the generating density of **B**. Note, that the injectivity of the cosine transform was shown by A.Aleksandrov .
Note also that one can prove Theorem 1 using the expansion of $h$ in the spherical harmonics.
We prove Theorem 1 just finding an inversion formula for (1).

# References


[1] P. Goodey, W. Weil, *Zonoids and generalizations*, In Handbook of convex geometry, ed. by P. M. Gruber and J. M. Wills, North Holland, Amsterdam, (1993), pp. 1297-326.

[2] R. Schneider, F.E. Schuster, *Rotation invariant Minkowski classes of convex bodies*. Mathematika 54 (2007), pp. 113.


## Random Schrodinger Operators With a Background Potential


H. Asatryan and W. Kirsch
(FernUniversität in Hagen, Germany)
hayk.asatryan@fernuni-hagen.de


On the space $L^2(\mathbb{R})$ we consider Schrödinger operators of the form

$$H_\omega := -\frac{d^2}{dx^2} + U + V_{\text{per}} + \sum_{k=-\infty}^{\infty} q_k(\omega) f(x-k) \quad (x \in \mathbb{R}) .$$

We assume that the background potential $U$, arising in the inverse problem of scattering, is a real-valued essentially bounded function and satisfies the relations

$$U(x) \;\to\; a^- \quad \text{as} \quad x \to -\infty, \qquad U(x) \;\to\; a^+ \quad \text{as} \quad x \to +\infty$$

with $a^{\pm} \in \mathbb{R}$. $V_{\text{per}}$ is assumed to be a 1-periodic essentially bounded function and $q_k$ $(k \in \mathbb{Z})$ are independent random variables with a common distribution $P_0$. We suppose that $f$, called the single-site potential, is a real-valued function and satisfies the estimate

$$|f(x)| \leqslant C\,(1+|x|)^{-\gamma} \quad (x \in \mathbb{R})$$

for some $\gamma > 1$. We assume for simplicity that supp $P_0$ is a compact subset of $\mathbb{R}$.

We study the influence of the background potential on the essential spectrum of the operator $H_\omega$ and obtain Anderson Localization (see [1]). Further, we prove the existence of the integrated density of states for $H_\omega$ and give a formula for it. The obtained results include generalizations of some known results of W. Kirsch, F. Martinelli, D. Damanik, G. Stolz and others (see [2], [3]).

# References


[1] Anderson P. W., *Absence of diffusion in certain random lattices.* – Phys. Rev., vol. 109, no. 5 (1958), pp. 1492–1505, DOI: 10.1103/PhysRev.109.1492

[2] Damanik D., Stolz G., *A continuum version of the Kunz-Souillard approach to localization in one dimension.* – J. reine angew. Math., no. 660 (2011), pp. 99–130, DOI: 10.1515/CRELLE.2011.070

[3] Kirsch W., Martinelli F., *On the density of states of Schrödinger operators with a random potential.* – J. Phys. A: Math. Gen., vol. 15, no. 7 (1982), pp. 2139–2156, DOI: 10.1088/0305-4470/15/7/025

# On Comparison in Distribution of Systems of Random Variables*

S. Astashkin
(Samara University, Russia)
astash56@mail.ru

We say that a system of random variables $\{f_i\}_{i=1}^{\infty}$ defined on a probability space $(\Omega, \Sigma, \mathbb{P})$ *is dominated in distribution* by a system of random variables $\{g_i\}_{i=1}^{\infty}$ defined on a probability space $(\Omega', \Sigma', \mathbb{P}')$ if there exists $C > 0$ such that for arbitrary $m \in \mathbb{N}$, $a_i \in \mathbb{R}$, $i = 1, 2, \ldots, m$, and $z > 0$ we have

$$\mathbb{P}\Big\{\omega \in \Omega : \Big|\sum_{i=1}^{m} a_i f_i(\omega)\Big| > z\Big\} \le C\mathbb{P}'\Big\{\omega' \in \Omega' : \Big|\sum_{i=1}^{m} a_i g_i(\omega')\Big| > C^{-1}z\Big\}.$$

The subject of this talk is the comparison of scalar or vector-valued sums of random variables with corresponding Rademacher sums, where the Rademacher functions are defined by

$$r_i(t) = \operatorname{sign}(\sin 2^i \pi t), \ t \in [0,1] \ (i = 1, 2, \ldots).$$

**Theorem 1.**[1] *Suppose that a sequence $\{g_i\}_{i=1}^{\infty}$ of random variables defined on a probability space $(\Omega, \Sigma, \mathbb{P})$ is dominated in distribution by the Rademacher sequence $\{r_i\}_{i=1}^{\infty}$. Then, there is $\tilde{C} > 0$ such that for every Banach space $F$ and all $m \in \mathbb{N}$, $x_i \in F$, $i = 1, 2, \ldots, m$, and $z > 0$ we have*

$$\mathbb{P}\Big\{\omega \in \Omega : \Big\|\sum_{i=1}^{m} x_i g_i(\omega)\Big\|_F > z\Big\} \le \tilde{C} m\Big\{t \in [0,1] : \Big\|\sum_{i=1}^{m} x_i r_i(t)\Big\|_F > \tilde{C}^{-1}z\Big\}$$

*($m$ is the Lebesgue measure).*

If the Rademacher sequence, conversely, is dominated by some sequence of random variables, the analogous result, in general, does not hold [1].

* This work was supported by the Ministry of Education and Science of the Russian Federation, project 1.470.2016/1.4 and by the RFBR Grant 17-01-00138.

We intend also to discuss connections of the above results with the results of the paper [2].

# References


[1] *Astashkin S. V.* On comparing systems of random variables with the Rademacher sequence, Izvestiya: Math. — 2017. — V. 81, no. 6, 1063–1079.

[2] *Bourgain J., Lewko M.* Sidonicity and variants of Kaczmarz's problem, Ann. Inst. Fourier, Grenoble — 2017. — V. 67, no. 3, 1321-1352.


## Hadamard Three Spheres Theorem for Quaternion-Valued Functions


K. Avetisyan
(Yerevan State University, Armenia)
avetkaren@ysu.am


Classical Hadamard three circles theorem for holomorphic functions on the complex plane asserts:

*Let $f(z)$ be a holomorphic function on the ring $\{z \in \mathbb{C} : r_1 \leq |z| \leq r_2\}$ and let $M_\infty(f;r)$ be the maximum of $|f(z)|$ on the circle $|z| = r$. Then $\log M_\infty(f;r)$ is a convex function of $\log r$.*

In this talk, we state a quaternion version of Hadamard three spheres theorem for monogenic functions in $\mathbb{R}^4$, namely:

*Let $f(x)$ be a (left) monogenic function on the spherical shell*

$$\{x \in \mathbb{R}^4 : r_1 < |x| < r_2,\ 0 \leq r_1 < r_2 \leq \infty\}.$$

*Then for any $p$ and $q$, $\frac{2}{3} \leq p < \infty$, $1 \leq q \leq \infty$, the integral means $M_q\big(|f|^p;r\big)$ and $\log M_1\big(\exp|f|^p;r\big)$ are convex functions of $r^{-2}$ on $(r_1,r_2)$. The index $2/3$ is principal and best possible.*

Earlier known results are for less general functions such as two-sided monogenic functions or Riesz-Stein-Weiss systems.

# On an Effective Solution of the Boundary Value Problem for One Improperly Elliptic Equation

S. Abelyan and A. Babayan
(National Polytechnic University of Armenia)
seyran.abelyan@gmail.com, barmenak@gmail.com

We consider in the unit disk $D$ with boundary $\Gamma = \partial D$ the following sixth order improperly elliptic equation

$$\sum_{k=0}^{6} A_k \frac{\partial^6 U}{\partial x^k \partial y^{6-k}} = 0, \quad (x,y) \in D, \tag{1}$$

where $A_k$ are such complex constants $(A_0 \neq 0)$, that the roots $\lambda_j$, $j = 1,\ldots,6$ of the characteristic equation

$$\sum_{k=0}^{6} A_k \lambda^{6-k} = 0 \tag{2}$$

satisfy the condition $\Im \lambda_k > 0$, $k = 1,2,\ldots,6$, that is the equation (1) is improperly elliptic. We suppose also, that all roots $\lambda_j \neq i$ are simple. We seek the solution $U$ of the equation (1), which belongs to the class $C^6(D) \bigcap C^{(2,\alpha)}(D \bigcup \Gamma)$ and on the boundary $\Gamma$ satisfies the Dirichlet boundary conditions

$$\left. \frac{\partial^k U}{\partial r^k} \right|_{\Gamma} = f_k(x,y), \;\; k = 0,1,2 \;\; (x,y) \in \Gamma, \tag{3}$$

Here $f_k$ $(k = 0,1,2)$ are prescribed functions on $\Gamma$, $\frac{\partial}{\partial r}$ is a derivative with respect to module of the complex number $(z = re^{i\theta})$. Class of the functions $f_k$ must be determined separately.

The problem (1), (3) for improperly elliptic equation (1) is not correct, (see [1]). In the papers [2,3] there were investigated some cases of properly elliptic equation (1).

In the present talk it was shown, that the homogeneous problem (1), (3) may have finitely many linearly independent solutions only in the case, when multiplicity of the root $i$ of the equation (1) is less than four. In this case it was shown that the boundary functions must be analytic in some annulus $\rho < |z| < 1$, where $\rho$ determined by the roots of the equation (2) in explicit form. The solutions of the homogeneous problem (1), (3) and conditions, provided solvability of the in-homogeneous problem (1), (3) are determined in explicit form.

# References


[1] Tovmasyan, N.E. Non-Regular Differential Equations and Calculation of the Electro - magnetic Fields. World Scientific Publishing Co. Pte. Ltd., 1998.

[2] A.O. Babayan, S.H. Abelyan. On a Dirichlet Problem for one Sixth Order Elliptic Equation. Reports of Enlarged Sessions of the Seminar of I. Vekua Institute of Applied Mathematics, Volume 31, 2017, pp. 7-10.

[3] S.H Abelyan, On an Effective solution of the Dirichlet Problem in the Unit Disc. Reports of NAS Armenii, Vol.118, No.1, 2017, pp. 15-19.


## Conditions for exponential decay of the variance of BLUE of mean for stationary sequences


N. M. Babayan and M. S. Ginovyan

(Russian-Armenian University, Armenia, and

Institute of Mathematics of NAS RA, Boston University, USA)

nmbabayan@gmail.com, ginovyan@bu.edu


Consider the following stochastic model $Y(t) = m + X(t)$, $t \in \mathbb{Z} := \{0, \pm1, \pm2, \ldots\}$, where $m$ is the constant unknown mean of $Y(t)$, and the noise $X(t)$ is assumed to be a zero-mean, wide-sense stationary process possessing a spectral density $f(\lambda)$, $\lambda \in [-\pi, \pi]$.

The problem of interest is the estimation of the unknown mean $m$ based on a random sample $\{Y(t), t = 0, 1, \ldots, n\}$. Of particular interest is the best linear unbiased estimator (BLUE) $\widehat{m}_{n,BLU}$ for $m$, that is, the estimator of the form $\widehat{m}_n = \sum_{k=0}^{n} c_k Y(k)$, where the weights $c_k$, $k = 0, 1, \ldots, n$, are chosen so that the variance $\mathrm{Var}\,(\widehat{m}_n) = E|\widehat{m}_n - m|^2$ is minimal under the condition $\sum_{k=0}^{n} c_k = 1$ (which is needed for unbiasedness of $\widehat{m}_n$).

We are interested here in the asymptotic behavior of the variance

$$\sigma_n^2(f) := \mathrm{Var}\,(\widehat{m}_{n,BLU}, f)$$

of $\widehat{m}_{n,BLU}$ as $n \to \infty$. Observe that using Kolmogorov's isometric isomorphism between the time- and frequency-domains: $X(t) \leftrightarrow e^{it\lambda}$, the variance $\sigma_n^2(f)$ can be represented in the form:

$$\sigma_n^2(f) = \min_{q_n \in \mathcal{Q}_n(1)} \int_{-\pi}^{\pi} \left| q_n(e^{i\lambda}) \right|^2 f(\lambda) d\lambda,$$

where $\mathcal{Q}_n(1)$ is the class of polynomials $q_n(z)$, $z \in \mathbb{C}$ of degree at most $n$, satisfying the condition $q_n(1) = 1$.

There is a substantial literature devoted to the asymptotic behavior of $\sigma_n^2(f)$ as $n \to \infty$ (see, e.g., Adenstedt [1], Vitale [2], and references therein). In all these papers were obtained conditions ensuring hyperbolic decay of the variance $\sigma_n^2(f)$, and, in general form, their results can be stated as follows: if $f(\lambda) \sim \lambda^\nu$ as $\lambda \to 0$, then $\sigma_n^2(f) \sim n^{-\nu-1}$ as $n \to \infty$.

The theorem that follows contains conditions for exponential decay of the variance $\sigma_n^2(f)$ as $n \to \infty$.

**Theorem.** *Let $\sigma_n^2(f) := Var\,(\widehat{m}_{n,BLU}, f)$. The following assertions hold:*
*(a) If the spectral density $f(\lambda)$ is positive almost everywhere in some vicinity of zero, then* $\lim_{n\to\infty} \sqrt[n]{\sigma_n(f)} = 1$.
*(b) If the spectral density $f(\lambda)$ vanishes almost everywhere for $|\lambda| < \alpha$, $0 < \alpha < \pi$, then $\sigma_n(f)$ decreases at least exponentially. More precisely, we have* $\lim_{n\to\infty} \sqrt[n]{\sigma_n(f)} \le \cos\frac{\alpha}{2}$.

It follows from assertion (a) of the theorem that a necessary condition for variance $\sigma_n^2(f)$ to decrease to zero exponentially as $n \to \infty$ is that the spectral density $f(\lambda)$ vanishes on a set of positive Lebesgue measure in any vicinity of zero. In particular, if $f(\lambda)$ vanishes only at the origin, then it is impossible to obtain exponential decay of $\sigma_n^2(f)$, no mater how high is the order of the zero of $f(\lambda)$ at the origin.

# References

[1] Adenstedt, R. K. (1974) On large-sample estimation for the mean of a stationary random sequence. *Ann. Statist.* 2, 1095-1107.

[2] Vitale, R. A. (1973) An asymptotically efficient estimate in time series analysis. *Q. Appl. Math.* 30, 421-440.


## Problems on Extremal Decomposition of the Complex Plane With Free Poles


A. Bakhtin and I. Denega

(Institute of Mathematics of NAS, Ukraine)

iradenega@gmail.com


The report is devoted to extremal problems in geometric function theory of complex variables associated with estimates of functionals defined on the systems of non-overlapping domains. Let $\mathbb{N}$, $\mathbb{R}$ be the sets of natural and real numbers, respectively, $\mathbb{C}$ be the complex plane, $\overline{\mathbb{C}} = \mathbb{C} \bigcup \{\infty\}$ be a one point compactification and $\mathbb{R}^+ = (0, \infty)$. Let $r(B, a)$ be the inner radius of the domain $B \subset \overline{\mathbb{C}}$ with respect to a point $a \in B$. Consider an extremal problem which was formulated in paper [1] in the list of unsolved problems and then repeated in monograph [2].

**Problem 1.**[1,2] Consider the product

$$r^{\gamma}(B_0, 0) \prod_{k=1}^{n} r(B_k, a_k), \tag{1}$$

where $B_0$, $B_1$,..., $B_n$ $(n \geq 2)$ are pairwise disjoint domains in $\overline{\mathbb{C}}$, $a_0 = 0$, $|a_k| = 1$, $k = \overline{1, n}$ and $0 < \gamma \leq n$. Show that it attains its maximum at a configuration of domains $B_k$ and points $a_k$ possessing rotational $n$-symmetry.

In 1988 V. Dubinin solved this problem for $\gamma = 1$ and $n \geq 2$. In 1996 L. Kovalev got the solution to this problem for $n \geq 5$ and subclass points systems satisfying condition $0 < \alpha_k \leq 2/\sqrt{\gamma}$, $k = \overline{1, n}$, where

$\alpha_k := \frac{1}{\pi} \arg \frac{a_{k+1}}{a_k}$, $\alpha_{n+1} := \alpha_1$, $k = \overline{1,n}$. In 2003 G. Kuz'mina studied this problem for $\gamma \in (0, 1]$.

**Problem 2.**[1,2] Show that the maximum of the product

$$[r(B_0, 0)\, r(B_\infty, \infty)]^\gamma \prod_{k=1}^{n} r(B_k, a_k),$$

where $\gamma \in \mathbb{R}^+$, $B_0$, $B_\infty$, $\{B_k\}_{k=1}^n$ are pairwise non-overlapping domains in $\overline{\mathbb{C}}$, $a_0 = 0$, $|a_k| = 1$, $k = \overline{1,n}$, $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{0,n}$, $\infty \in B_\infty \subset \overline{\mathbb{C}}$, is achieved for some configuration of the domains $B_k$, $B_\infty$ and points $a_k$, $\infty$, $k = \overline{0,n}$, which are having $n$-fold symmetry.

For $\gamma = \frac{1}{2}$ and $n \geq 2$ the problem 2 was solved in 1988 by V. Dubinin. In 2001 G. Kuz'mina showed that the Dubinin's estimate is correct when $\gamma \in \left(0, \frac{n^2}{8}\right]$, $n \geq 2$.

**Problem 3.**[1,2] Let $a_0 = 0$, $|a_k| = 1$, $k = \overline{1,n}$, $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{0,n}$, where $B_0, \ldots, B_n$ are pairwise non-overlapping domains and $B_1, \ldots, B_n$ are symmetric domains with respect to the unit circle. Find the exact upper bound of the product (1).

In 2000 L. Kovalev solved the problem 3 for $n \geq 2$ and $\gamma = 1$.

In the report a review of the latest results obtained in the above-mentioned problems will be presented.

# References


[1] Dubinin V.N. *Symmetrization method in geometric function theory of complex variables*. Russian Math. Surveys. 1994. V. 1. no 1. P. 1–79.

[2] Dubinin V.N. *Condenser capacities and symmetrization in geometric function theory*. Birkhäuser/Springer. Basel. 2014.

[3] Bakhtin A.K., Bakhtina G.P., Zelinskii Yu.B. *Topological-algebraic structures and geometric methods in complex analysis*. Zb. prats of the Inst. of Math. of NASU. 2008. (in Russian)

[4] I.V. Denega, Ya.V. Zabolotnii. *Estimates of products of inner radii of non-overlapping domains in the complex plane*. Complex Variables and Elliptic Equations. 2017. V. 62, no. 11, pp. 1611–1618.

[5] A. Bakhtin, L.Vygivska, and I. Denega. *N-Radial Systems of Points and Problems for Non-Overlapping Domains*. Lobachevskii Journal of Mathematics. 2017. V. 38, no. 2, pp. 229–235.

# Convergence Acceleration of the Trigonometric Expansions

R. Barkhudaryan and A. Poghosyan
(Institute of Mathematics of NAS, Armenia)
rafayel@instmath.sci.am, arnakp@gmail.com

We consider the problem of function reconstruction by its finite number of Fourier coefficients (continuous or discrete). In case of smooth, but non-periodic functions, the drawback of the classical trigonometric expansions is hidden in the Gibbs phenomenon. Different approaches are known for elimination of the phenomenon and convergence acceleration of trigonometric expansions.

First, we discuss a polynomial subtraction approach invented by Krylov [1] and independently considered by Lanczos [2]. The polynomial represents the discontinuities in the function and some of its first derivatives (jumps) and practical realization of the approach is connected with accurate approximation of the laters [3]. In a series of papers, the accuracy of jumps approximation and the convergence of corresponding trigonometric expansions were explored. It was proved that the expansions with approximate jumps are more accurate than with the exact ones. This interesting behavior of the Krylov-Lanczos-Eckhoff approach is known as auto-correction phenomenon ([4–6]).

Second, we discuss an approach based on rational correction functions ([7–9]). Rational corrections contain unknown parameters which determination is a crucial problem for realization of the rational approximations. One possible choice leads to Fourier-Pade approximations and interpolations. Another choice leads to optimal rational expansions.

## References

[1] Krylov, A. On approximate calculations: Lectures delivered in 1906. Tipolitography of Birkenfeld, St. Petersburg, 1907.

[2] Lanczos, C. Discourse on Fourier Series, Oliver and Boyd, Edinburgh, 1966.

[3] Eckhoff, K. S. Accurate and efficient reconstruction of discontinuous functions from truncated series expansions. Math. Comp., 61(204)(1993), 745–763.

[4] Barkhudaryan, A.; Barkhudaryan, R.; Poghosyan, A. Asymptotic behavior of Eckhoff's method for Fourier series convergence acceleration. Anal. in Theory and Appl., **23**(3)(2007), 228–242.

[5] Poghosyan, A. On an auto-correction phenomenon of the Krylov–Gottlieb–Eckhoff method. IMA Journal of Numerical Analysis, **31**(2)(2011), 512–527.

[6] Poghosyan, A. On an autocorrection phenomenon of the Eckhoff's interpolation. Australian J. of Math. Anal. and Appl., **9**(1)(2012), Article 19, 1–31.

[7] Nersesyan, A. B. Bernoulli-type quasipolynomials and the acceleration of the convergence of Fourier series of piecewise-smooth functions. (Russian) Dokl. Nats. Akad. Nauk Armen. 104 (2004), no. 4, 273–279.

[8] Poghosyan, A. On a convergence of the Fourier-Pade approximation. Armenian Journal of Mathematics, **4**(2)(2012), 49–79.

[9] Poghosyan, Arnak V.; Bakaryan, Tigran K. Optimal rational approximations by the modified Fourier basis. Abstr. Appl. Anal. 2018, Art. ID 1705409, 21 pp.

## Approximation by Sums of Shifts in $L_p$ Spaces

P. Borodin
(Moscow State University, Russia)
pborodin@inbox.ru

Theorem 1. There is a function $f\colon \mathbb{R} \to \mathbb{R}$ such that the sums

$$\sum_{k=1}^{n} f(x - a_k), \qquad a_k \in \mathbb{R}, \quad n = 1, 2, \dots,$$

of its shifts are dense in all real spaces $L_p(\mathbb{R})$ for $2 \leq p < \infty$ and also in the real space $C_0(\mathbb{R})$ of continuous functions tending to zero at $\pm\infty$.

Theorem 2. *There is an element $v$ in the real space $l_2(\mathbb{Z})$ such that the sums*

$$\sum_{k=1}^{n} T^{j_k}(v), \qquad j_k \in \mathbb{Z}, \quad n = 1, 2, \dots,$$

*of its shifts are dense in all real spaces $l_p(\mathbb{Z})$, $2 \leq p < \infty$, and also in the real space $c_0(\mathbb{Z})$.*

These theorems are based on certain results due to S.V. Konyagin on convergence to zero of trigonometric polynomials with positive integer coefficients.

## Nehari's Theorem For Convex Domain Hankel and Toeplitz Operators in Several Variables

M. Carlsson
(Lund University, Sweden)
Marcus.carlsson@math.lu.se

We prove Nehari's theorem for integral Hankel and Toeplitz operators on simple convex polytopes in several variables. A special case of the theorem, generalizing the boundedness criterion of the Hankel and Toeplitz operators on the Paley–Wiener space, reads as follows. Let $\Xi = (0,1)^d$ be a $d$-dimensional cube, and for a distribution $f$ on $2\Xi$, consider the Hankel operator

$$\Gamma_f(g)(x) = \int_{\Xi} f(x+y)g(y)\,dy, \quad x \in \Xi.$$

Then $\Gamma_f$ extends to a bounded operator on $L^2(\Xi)$ if and only if there is a bounded function $b$ on $\mathbb{R}^d$ whose Fourier transform coincides with $f$ on $2\Xi$. This special case has an immediate application in matrix extension theory: every finite multi-level block Toeplitz matrix can be boundedly extended to an infinite multi-level block Toeplitz matrix. In particular, block Toeplitz operators with blocks which are themselves Toeplitz, can be extended to bounded infinite block Toeplitz operators with Toeplitz blocks.

# Approximation of Bergman Kernels by Rational Function With Fixed Poles

S. Chaichenko
(Donbas State Pedagogical University, Ukraine)
s.chaichenko@gmail.com

Let $\mathbf{a} := \{a_k\}_{k=0}^{\infty}$ be a sequence of points in the unit circle $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ of the complex plane $\mathbb{C}$, where we allow points of finite and infinite multiplicity. The Takenaka-Malmquist system of functions, generated by the sequence $\mathbf{a}$, is the system $\varphi := \{\varphi_k\}_{k=0}^{\infty}$ of functions $\varphi_k$ of the form [1]:

$$\varphi_0(z) := \frac{\sqrt{1-|a_0|^2}}{1-\overline{a_0}z}, \ \varphi_k(z) := \frac{\sqrt{1-|a_k|^2}}{1-\overline{a_k}z} \prod_{j=0}^{k-1} \frac{-|a_j|}{a_j} \frac{z-a_j}{1-\overline{a_j}z}, \ k = 1, 2, \dots, \tag{1}$$

where for $a_j = 0$ we assume $|a_j|/a_j = -1$.

In the theory of Bergman spaces functions of the form (see, for example, [2, p. 6])

$$\mathcal{K}_\alpha(z; w) = \frac{1}{(1-z\overline{w})^{2+\alpha}}, \quad \alpha = 0, 1, \dots, \quad z, w \in \mathbb{D},$$

play an essential role. That functions are called the (weighted) Bergman kernels of $\mathbb{D}$.

For given $n$ $(1 \le n < \infty)$ we denote by $\mathcal{R}(n)$ the set of rational functions of the form

$$R_n(x) = c_0 + \sum_{m=1}^{n} c_m \varphi_{m-1}(x), \quad c_m \in \mathbb{C}.$$

**Theorem.** *On the set of functions $\mathcal{R}(n)$ the minimum of the functional*

$$\mu_\alpha(R_n) := \int_{\mathbb{T}} \left| \mathcal{K}_\alpha(x; w) - \frac{R_n(x)}{(1-x\overline{w})} \right|^2 d\sigma(x)$$

*is realized on the function*

$$r_{\alpha,n}(x;w) = (1 - x\overline{w})S_{n+1}(\mathcal{K}_\alpha)(x;w) \in \mathcal{R}(n),$$

*where* $S_{n+1}(\mathcal{K}_\alpha)(x;w)$ *is the partial sum of the order* $n+1$ *of Fourier series of function* $\mathcal{K}_\alpha(x;w)$ *on the system (1),* $a_n = a_{n-1} = \ldots = a_{n-\alpha} \equiv w \in \mathbb{D}$*, and*

$$\inf_{R_n \in \mathcal{R}(n)} \mu_\alpha(R_n) = \mu_\alpha(r_{\alpha,n}) = \frac{|w|^{2\alpha+2}}{(1-|w|^2)^{2\alpha+3}} |B_{n-\alpha}(w)|^2.$$

# References


[1] Walsh J.L., Interpolation and Approximation by Rational Functions in the Complex Domain. — New York: American Mathematical Society Colloquium Publications, vol. 20, 1935.

[2] H. Hedenmalm, B. Korenblum, K. Zhu. Theory of Bergman Spaces. — New York-Berlin-Heidelberg: Springer-Verlag, 2000.


## On Exact Recovery of Dirac Ensembles From Projections Onto Polynomial Spaces


Sh. Dekel
(Tel-Aviv University, Israel)
shaidekel6@gmail.com


We are given the projection of a superposition of Diracs onto a finite dimensional polynomial space over a manifold (e.g. trigonometric polynomials, algebraic polynomials, spherical harmonics) and we wish to recover the signal exactly and in particular, the locations of the knots. We will show that under a separation condition on the support of the unknown signal, there exists a unique solution through TV minimization over the space of Borel measures. By duality, this problem is strongly connected to the constructions of interpolating polynomials under the restriction that the interpolation points are the extremal points of the polynomials.


Joint work with Tamir Bendory (Princeton) and Arie Feuer (Technion).

# References


[1] E. Candes and C. Fernandez-Granda. Towards a mathematical theory of super-resolution. Communications on Pure and Applied Mathematics 67 (2014), 906-956.

[2] T. Bendory, S. Dekel and Arie Feuer, Exact recovery of non-uniform splines from the projection onto spaces of algebraic polynomials, Journal of Approximation Theory 182 (2014), 7-17.

[3] T. Bendory, S. Dekel and Arie Feuer, Super-resolution on the sphere using convex optimization, IEEE Transactions on signal processing 63 (2015), 2253-2262.

[4] T. Bendory, S. Dekel and Arie Feuer, Exact recovery of Dirac ensembles from the projection onto spaces of spherical harmonics, Constructive Approximation 42 (2015), 183-207.

[5] T. Bendory, S. Dekel and Arie Feuer, Robust recovery of streams of pulses using convex optimization, Journal of mathematical analysis and applications 442 (2016), 511-536.


## Approximation of Log-Convex Weights by Integral Means of Holomorphic Functions


E. Dubtsov
(St. Petersburg Department of V.A.Steklov Mathematical Institute, Russia)
dubtsov@pdmi.ras.ru


Let $\mathcal{H}ol(B_d)$ denote the space of holomorphic functions on the unit ball $B_d$ of $\mathbb{C}^d$, $d \geq 1$. Given a log-convex strictly positive weight $w(r)$ on $[0,1)$, we construct a function $f \in \mathcal{H}ol(B_d)$ such that the standard integral means $M_p(f,r)$ and $w(r)$ are equivalent for any $0 < p \leq \infty$. Also, we obtain similar results related to volume integral means.

# On solvability of Dirichlet problem for second order elliptic equation

V. Dumanyan
(Yerevan State University, Armenia)
duman@ysu.am

We study the solvability of the Dirichlet problem for the general second-order elliptic equation

$$-div(A(x)\nabla u)+(\bar{b}(x),\nabla u)-div(\bar{c}(x)u)+d(x)u=f(x)-divF(x), \quad x\in Q, \tag{1}$$

$$u|_{\partial Q}=u_0, \tag{2}$$

in a bounded domain $Q \subset R_n$, $n \geq 2$, $\partial Q \in C^1$, with boundary function $u_0$ in $L_2(\partial Q)$. We assume that $f$ and $F = (f_1, \dots, f_n)$ are in $L_{2,loc}(Q)$ and that the symmetric matrix $A(x) = (a_{ij}(x))$, consisting of real-valued measurable functions, satisfies the condition

$$\gamma|\xi|^2 \leq \sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j = (\xi, A(x)\xi) \leq \gamma^{-1}|\xi|^2$$

for all $\xi = (\xi_1, \dots, \xi_n) \in R_n$ and almost all $x \in Q$, where $\gamma$ is a positive constant. We also assume that the coefficients $\bar{b}(x) = (b_1(x), \dots, b_n(x))$, $\bar{c}(x) = (c_1(x), \dots, c_n(x))$ and $d(x)$ are measurable and bounded on any strictly interior subdomain of $Q$.

Precise (in terms of the growth order) constraints are obtained on the growth near the boundary of the considered bounded domain $Q$ for the lower order coefficients of the equation for the solution from the space $W^1_{2,\,loc}(Q)$ is $(n-1)$-dimensional continuous.

Necessary and sufficient conditions for the existence of an $(n-1)$-dimensional continuous solution of problem (1), (2) were obtained in terms of the inner product in a special Hilbert space. It was also proved that the solvability conditions for the considered problem have a form similar to the solvability conditions in the usual generalized setting (in the space $W_2^1(Q)$). In particular, it was proved that if the homogeneous problem (with zero boundary function and the right side) has no non-zero solutions in $W_2^1(Q)$, then for any boundary function $u_0 \in L_2(\partial Q)$ and any right side from an appropriate functional space there exists a solution for the non-homogeneous problem. This solution belongs to Gushchin space $C_{n-1}(\bar{Q})$ and the following estimate holds

$$\|u\|^2_{C_{n-1}(\bar{Q})} + \int\limits_Q r\,|\nabla u|^2 dx \leq$$

$$\leq C\Big( \|u_0\|^2_{L_2(\partial Q)} + \int\limits_Q r^3\,(1+|\ln r|)^{\frac{3}{2}}\, f^2 dx + \int\limits_Q r\,(1+|\ln r|)^{\frac{3}{2}}\,|F|^2 dx\Big),$$

where $r$ is the distance from a point $x \in Q$ to the boundary $\partial Q$ and the constant $C$ does not depend on $u_0$, $f$, $F$.

For some natural constraints on the lower order coefficients and for the right sides from $W_2^{-1}(Q)$ obtained necessary and sufficient conditions for solvability of the problem in $C_{n-1}(\bar{Q})$ can be stated in a simpler form - in terms of the original problem. It was also proved that if the boundary function has an extension to $Q$ that belongs to $W_2^1(Q)$ then the solution in $C_{n-1}(\bar{Q})$ is at the same time a solution in $W_2^1(Q)$ and in such a case the solvability conditions in $C_{n-1}(\bar{Q})$ coincide with those in $W_2^1(Q)$.

## On Pointwise Universality of Partial Sums Of Fourier Series by the Generalized Walsh System

S. Episkoposian and J. Meuller
(National Politechnic University of Armenia and Universität Trier, Germany)
sergoep@ysu.am, jmueller@uni-trier.de

Let $a$ denote a fixed integer, $a \geq 2$ and put $\omega_a = e^{\frac{2\pi i}{a}}$.

Now we give the definitions of the generalized Rademacher and Walsh systems.

**Definition 1.** The Rademacher system of order $a$ is defined by

$$\varphi_0(x) = \omega_a^k \;\; if \;\; x \in \left[\frac{k}{a}, \frac{k+1}{a}\right), \;\; k = 0, 1, ..., a-1,$$

and for $n \geq 0$

$$\varphi_n(x+1) = \varphi_n(x) = \varphi_0(a^n x).$$

**Definition 2.** The generalized Walsh system of order $a$ is defined by

$$\psi_0(x) = 1,$$

and if $n = \alpha_1 a^{n_1} + ... + \alpha_s a^{n_s}$ where $n_1 > ... > n_s$, then

$$\psi_n(x) = \varphi_{n_1}^{\alpha_1}(x) \cdot ... \cdot \varphi_{n_s}^{\alpha_s}(x).$$

The basic properties of the generalized Walsh system of order $a$ are obtained by H.E. Chrestenson, J. Fine, C. Vatari, N. Vilenkin and others. Let's denote the generalized Walsh system of order $a$ by $\Psi_a$. Note that $\Psi_2$ is the classical Walsh system.

For any fixed $a \geq 2$ we consider the space $\mathcal{C}_a[0,1)$ - the class of all functions defined on $[0,1)$, continuous at all points of $[0,1) \setminus \mathcal{Q}_a$ and continuous on the right at the points $\mathcal{Q}_a = \{\frac{m}{a^k}, \;\; 0 \leq m \leq a^k - 1, \;\; k = 0, 1, 2, ....\}$ and problems of pointwise universality of partial sums of Fourier series of generalized Walsh system $\Psi_a$, $a \geq 2$ of functions in the class $\mathcal{C}_a[0,1)$.

The norm in this space is defined as an ordinary uniform norm, i.e,

$$||f||_{\mathcal{C}_a} = \sup_{x \in [0,1)} |f(x)|, \;\; f(x) \in \mathcal{C}_a[0,1).$$

Next, let $E \subset [0,1)$ be a countable set and $\Lambda \subset \mathcal{N}$ be a set of indices. We denote by $\mathcal{C}^E$ the set of all functions $h(x)$ defined on $E$ and taking complex values, i.e, $h(x_k) = y_k \in \mathcal{C}$, $k = 1, 2, ...$, and by $\{S_n(f)\}$ we denote the sequence of partial sums of the Fourier series $f(x) \in \mathcal{C}_a[0,1)$, by the generalized Walsh system.

**Definition.** *We say that the sequence of partial sums of the Fourier series of* $f(x) \in \mathcal{C}_a[0,1)$ *is pointwise universal in* $\mathcal{C}^E$*, if for any function* $h \in \mathcal{C}^E$ *there is* $n_k \in \Lambda$ *such that*

$$S_{n_k}(f)(x_j) \to h(x_j), \text{ as } k \to \infty, \text{ for all } j = 1, 2, ...$$

The following is true :

**Theorem .** *For any countable set* $E \subset [0,1)$ *there exists a dense* $G_\delta$ *- set of* $\mathcal{M} \subset \mathcal{C}_a[0,1)$ *with the following property: the sequence of partial sums of Fourier series of any function in* $\mathcal{M}$ *is pointwise universal in* $\mathcal{C}^E$*.*

## On the Convergence of the Negative Order Cesaro Means of Fourier Series With Respect to the Trigonometric and Walsh Systems

L. Galoyan
(Yerevan State University, Armenia)
levongaloyan@mail.ysu.am

In this talk we present certain results concerning the convergence (and divergence) of negative order Cesaro means of Fourier series with respect to the trigonometric and Walsh systems. In particular, we will consider the possibility of constructing continuous functions with divergent Cesaro means, and also the convergence of Cesaro means in $L^p-$metrics of Fourier series with monotonic coefficients.

It is well-known that the partial sums $S_n(f;x)$ of the trigonometric Fourier series of function $f(x) \in L^1[-\pi,\pi]$ converges to that function in $L^p-$ metric for each $p \in (0,1)$. Therefore, any trigonometric Fourier series of summable function possesses an almost everywhere convergent subsequence of partial sums. The same result holds also for the Walsh system. In connection with this, D. E. Menshov stated the following problem: will this statement be true if the ordinary convergence would be replaced by the negative order Cesaro summation methods? In other words, does there exist an almost everywhere convergent subsequence of the negative order Cesaro means of the Fourier series of any summable function? Menshov has shown that the answer to the last question is negative. More precisely, he proved that there exists a function $f_0 \in L^2[-\pi,\pi]$, such that for any $\alpha \in (-1,-1/2)$ and for any increasing sequence of natural numbers $\{m_\nu\}_{\nu=1}^{\infty}$ the sequence of $\alpha-$order Cesaro means $\sigma_{m_\nu}^{\alpha}(f_0,x)$ of Fourier series of $f_0$ is diverges on a set of positive measure.

In [1] we strengthened this result in the following way (see also [2]).

**Theorem A** There exists a function $f_0 \in C[-\pi,\pi]$, such that for an arbitrary increasing sequence of natural numbers $\{m_\nu\}_{\nu=1}^{\infty}$ and for each $\alpha \in (-1,-1/2)$ we have

$$|\{x \in [-\pi,\pi] : \limsup_{\nu\to\infty} \left|\sigma_{m_\nu}^{\alpha}(f_0,x)\right| = +\infty\}| > 0.$$

We have also proved that the last result holds also in the case of the Walsh systems.

**Remark.** It can be shown that the $\alpha-$order Cesaro means of Fourier series in the trigonometric and Walsh systems of any function from $L^2$ contain a convergent subsequence for each $\alpha \in (-1/2,0)$, consequently, the order -1/2 here cannot be improved. In [3] it is proved that

**Theorem B** Let $p > 1$, then for each $f \in L^p(0,1), p > 1$ the $\alpha-$order Cesaro means of the Fourier series of $f$ with monotonic Fourier coefficients converges in $L^p$ metric if $\alpha \in (1/p-1,0)$. If $\alpha < 1/p-1$, then there exists a function from $L^p(0,1)$ with monotonic Fourier coefficients, the $\alpha-$order Cesaro means of Walsh-Fourier series of which diverge in $L^p$ metric.

The proof of a similar result in the case of a trigonometric system was represented in [4].

# References


[1] M. Grigoryan, L. Galoyan, On the uniform convergence of negative order Cesaro means of Fourier series, Journal of Mathematical Analysis and Applications, Vol.434, iss.1, 2016, pp. 554-567.

[2] L. Galoyan, M. Grigoryan, A. Kobelyan, Convergence of Fourier series in classical systems, Sbornik: Mathematics, 2015, 206:7, pp. 941-979.

[3] L. Galoyan, On convergence of negative order Cesaro means of the Fourier-Walsh series in $L_p, p > 1$ metrics, Journal of Contemporary Mathematical Analysis 47(3), 2012, pp. 35-54.

[4] L. Galoyan, On Lp-convergence of Cesaro means for Fourier series with monotonic coefficients, Russian Mathematics (Izvestiya VUZ. Matematika), 2016, 60:2, pp. 19-24.


## On the Castillon - Cramer problem


L. Gevorgyan

(National Polytechnic University of Armenia)

levgev@hotmail.com


From Wikipedia, the free encyclopedia "In geometry, the Cramer - Castillon problem is a problem stated by the Swiss mathematician Gabriel Cramer solved by the Italian mathematician, resident in Berlin, Jean de Castillon in 1776. Given a circle $\Gamma$ and three points $A, B, C$ in the same plane and not on $\Gamma$, to construct every possible triangle inscribed in $\Gamma$ whose sides (or their elongations) pass through *A*,*B*,*C* respectively. Centuries before, Pappus of Alexandria had solved a special case: when the three points are collinear. But the general case had the reputation of being very difficult. After the geometrical construction of Castillon, Lagrange found an analytic solution, easier than Castillon's. In the beginning of the 19th century, Lazare Carnot generalized it to $n$ points $A_1, A_2, \cdots, A_n$. "

The problem may have no solution, a unique solution, two solutions and continuum solutions.

In this report the necessary and sufficient condition of the solvability of the Castillon - Cramer problem is proved and some sufficient conditions are formulated.

# References


[1] Ostermann A., Gerhard W., Geometry by Its History, Springer Verlag, 2012.

[2] Gerhard W., The Cramer-Castillon problem and Urquhart's "most elementary " theorem, Elem. Math. 61 (2006) 58-64.

[3] Stark M., Castillon's problem, WFNMC-4 Conference -Melbourne 2002, Abstract.


## On the Uniqueness of Double Franklin Series


G. Gevorkyan

(Yerevan State University, Armenia)

ggg@ysu.am


Let $\{f_n(x)\}_{n=0}^{\infty}$ be the Franklin system on $[0,1]$. In [1] it was proved that if the Franklin series $\sum_{n=0}^{\infty} a_n f_n(x)$ converges everywhere to an everywhere finite integrable function $f$, then the series is the Fourier-Franklin series of $f$. It is also known that the series $\sum_{n=0}^{\infty} f_n(t_0) f_n(x)$ converges to zero everywhere, except the point $t_0$. So the set $\{t_0\}$, consisting of one point, is not a uniqueness set for Franklin series.

**Theorem 1.** *Let the series $\sum_{n=0}^{\infty} a_n f_n(x)$ converges in measure to an integrable function $f$ and everywhere, maybe except of points $x_i$, $i = 1, 2, ..., m$, satisfies*

$\sup_N \left|\sum_{n=0}^{N} a_n f_n(x)\right| < \infty$. *Then the following representation is true:*

$$a_n = (f, f_n) + \sum_{i=1}^{m} (A_i f_n(x_i) + B_i f_n'(x_i + 0) + C_i f_n'(x_i - 0)),$$

*where $A_i$, $B_i$, $C_i$ are some constants.*

Let $E^0 = \bigcup_{n=0}^{\infty} \{x \in [0,1] : f_n(x) = 0\}$. Using the Theorem 1 and methods, developed in [2], we prove the following theorem.

**Theorem 2.** *Let $S_{M,N}(x,y)$ be the rectangular partial sums of double Franklin series $\sum_{m,n=0}^{\infty} a_{mn} f_m(x) f_n(y)$. Suppose*

$$\lim_{M,N \to \infty} S_{M,N}(x,y) = f(x,y), \quad \text{if } (x,y) \notin \mathcal{G},$$

*where $f$ is an everywhere finite integrable function and*

$$\mathcal{G} = \bigcup_{i=1}^{k} X_i \times \{y_i\}, \quad \overline{[0,1] \backslash X_i} = [0,1] \text{ and } ([0,1] \backslash X_i) \backslash E^0 \neq \emptyset.$$

*Then the series is the Fourier-Franklin series of function $f$. In particular, every finite set is a Valle-Poussen's type set for double Franklin series.*

# References


[1] G.G. Gevorkyan *Uniqueness theorems of Franklin series, converging to integrable functions*, Mat. Sb. 209:6 (2018), 25-46.

[2] L.D. Gogoladze, *On the theorem of reconstructing the coefficients of convergent multiple function series*, Izv. RAN. Ser. Mat., 72:2 (2008), 83–90 (in Russian). English translation: Izv. Math., 72:2 (2008), 283–290.

## Subsequences of Logarithmic Means of Walsh-Fourier Series *

U. Goginava
( Tbilisi State University, Georgia)
zazagoginava@gmail.com

We discuss some convergence and divergence properties of subsequences of logarithmic means of Walsh-Fourier series . We give necessary and sufficient conditions for the convergence regarding logarithmic variation of numbers.

## On the quasi-greedy constant of the Haar subsystems in $L_1(0,1)^d$

S. Gogyan and N. Srapionyan
(Institute of Mathematics of NAS Armenia, Yerevan State University, Armenia)
gogyan@instmath.sci.am, nerses.srapionyan@gmail.com

It is known that the Haar system is not a quasi-greedy basis in $L_1(0,1)$. However, there are Haar subsystems that form quasi-greedy bases in the closure of their linear span, see [1]. This result is generalized for a multivariate Haar system in $L_1(0,1)^d$ in [3], where the author obtains the following estimation for the quasi-greedy constant,

$$\|G_n\| \leq 2^{d(H+2)},$$

*The author is supported by Shota Rustaveli National Science Foundation grant 217282. Operators of Fourier analysis in some classical and new function spaces

where $H$ is the length of the maximal chain of the subsystem. In this paper, we improved the known exponential dependence by linear estimation, and show that

$$\|G_n\| \leq 2^d H + 1.$$

# References


[1] S. Dilworth, D. Kutzarova, P. Wojtaszczyk On approximate l1 systems in Banach spaces. *J. Approx. Theory* 114(2):214-241, 2002.

[2] S. Gogyan Greedy algorithm with regard to Haar subsystems. *East J. Approx.*, 00(00):221-236, 2005.

[3] S. Gogyan Quasi-greedy property of subsystems of the multivariate Haar system. *Izvestiya: Mathematics.*, 80(3):481488, 2016.

[4] V. Temlyakov Greedy approximation. *Acta Numberica.*, 17:235-409, 2008.

[5] P. Wojtaszczyk Greedy algorithms for general biorthogonal systems. *J. Approx Theory..*, 00(00):293-314, 2000.


## On the Unconditional Quasi-Basis Property of the Faber–Schauder System


M. Grigoryan and V. Krotov

(Yerevan State University, Armenia, Belarusian State University, Belarus)

gmarting@ysu.am, vgkrotov@mail.ru


Let $\{\Phi_n\}_{n=0}^{\infty}$ be the Faber–Schauder system that forms a basis in the space $C[0,1]$ of continuous functions on $[0,1]$. That is, for any function $f \in C[0,1]$ there exists a unique series of the form

$$\sum_{n=0}^{\infty} A_n(f)\Phi_n$$

(the Fourier–Faber–Schauder series) convergent to $f$ in $C[0,1]$. Denote by $\mathrm{Spec}(f)$ the set of indices $n$ such that $A_n(f) \neq 0$.

**Theorem.** *Let the sequence* $\{b_n\} \subset \mathbb{R}$ *be decreasing to 0 and*

$$\sum_{n=1}^{\infty} \frac{b_n}{n} = \infty.$$

*Then there exists a sequence of signs* $\{\varepsilon_n = \pm 1\}$ *such that for any* $\delta > 0$ *there exists a measurable set* $E \subset [0,1]$, $|E| > 1-\delta$, *such that for any function* $f \in C[0,1]$ *we can find a function* $\widetilde{f} \in C[0,1]$ *with the following properties:*

1) $f(x) = \widetilde{f}(x)$ *for all* $x \in E$,

2) $A_n(\widetilde{f}) = \varepsilon_n b_n$ *for all* $n \in \mathrm{Spec}(\widetilde{f})$,

3) *the Fourier–Faber–Schauder series* $\sum_{n=0}^{\infty} A_n(\widetilde{f})\Phi_n$ *of* $\widetilde{f}$ *converges unconditionally in* $C[0,1]$.

**Corollary.** *For any* $0 < \delta < 1$ *there exists a measurable set* $E \subset [0,1]$ *with measure* $|E| > 1-\delta$, *such that for any function* $f \in C[0,1]$ *we can find a function* $\widetilde{f} \in C[0,1]$ *that coincides with* $f$ *on* $E$, *and the Fourier–Faber–Schauder series of* $\widetilde{f}$ *converges unconditionally in* $C[0,1]$.

A basis in a Banach space is called unconditional if it remains a basis after any permutation. It is known that in the space $C[0,1]$ there are no unconditional bases (S.Karlin). In particular, $\{\Phi_n\}_{n=0}^{\infty}$ is not an unconditional basis. The property of this system, formulated in the Corollary, is naturally called «unconditional quasi-basis property». Not every basis in the space $C[0,1]$ has this property, for example, the Franklin system.

# References


[1] Grigoryan M.G., Sargsyan A.A., *Unconditional C-strong property of Faber-Schauder system*, Journal of Math.Anal.Appl., 352:2 (2009), p. 718-723.

[2] Grigoryan M.G., Krotov V.G. *Luzin' correction theorem and the coefficients of Fourier expansions in the Faber-Schauder system*, Math. Notes, 93:1-2 (2013), p. 217-223.

[3] Galoyan L.N., Grigoryan M.G., Kobelyan A.Kh., *Convergence of Fourier series in classical systems*, Sbornik Math, 206:7 (2015), 941-979.

# Functions, Universal With Respect to Classical Systems

M. Grigoryan
(Yerevan State University, Armenia)
gmarting@ysu.am

In this talk we present some new results.

**Theorem 1.** There exist a (universal) function $U \in L^1[0,1)$ and a sequence signs $\delta_k = \pm 1$ such that the series $\sum_{|k|=0}^{\infty} \delta_k c_k(U) e^{i2\pi kx}$ is universal in the class of all *measurable functions* in the common sense,

*that is, for each function $f \in M$ one can find a sequence $\{N_m\}_{m=1}^{\infty} \nearrow$ such that*

$$\lim_{m\to\infty} \sum_{0\le|k|\le N_m} \delta_k c_k(U) e^{i2\pi kx} = f(x) \quad \textit{almost everywhere on } [0,1],$$

*where*

$$c_k(U) = \int_0^1 U(x) e^{-i2\pi kx} dx, k = 0, \pm1, \pm2, \dots .$$

**Theorem 2.** There exists a (universal) function $U \in L^1[0,1)$ with strictly decreasing Fourier-Walsh coefficients and Fourier-Walsh series converging in the $L^1[0,1)$ norm, with the following property:

1) one can find a sequence of signs (numbers) $\delta_k = \pm 1$ such that the series $\sum_{k=0}^{\infty} \delta_k d_k(U) W_k(x)$ is universal in $L^p[0,1)$ for all $p \in (0,1)$ in the common sense,

2) one can find a sequence of (signs) numbers $\varepsilon_k = \pm 1$ such that the series $\sum_{k=0}^{\infty} \varepsilon_k d_k(U) W_k(x)$ is universal in $L^p[0,1)$ for all $p \in [0,1)$ in the sense of rearrangements, that is, for each function $f \in L^p[0,1)$, $p \in [0,1)$, one can find a permutation $\{\sigma(k)\}_{n=1}^{\infty}$ of nonnegative integers, such that the

series $\sum_{k=0}^{\infty} \varepsilon_{\sigma(k)} d_{\sigma(k)}(U) W_{\sigma(k)}(x)$ converges to $f$ in $L^p[0,1)$ metric (where $d_k(U) = \int_0^1 U(x) W_k(x) dx$ ).

**Remark**. According to the Kolmogorov's theorem (the Fourier series of any integrable function converges in $L^p[0,1)$ , $p \in (0,1)$ ) there is no integrable function which Fourier series with respect to the trigonometric system is universal in the common sense.

# References


[1] M.G. Grigoryan, On the universal and strong property $(L^1, L^\infty)$ related to Fourier-Walsh series, Banach Journal of Math.Analysis,11,2017, no.3, 698-712.

[2] M. G. Grigoryan, A. A. Sargsyan, On the universal function for the class , Journal of Func. Anal., 270, 8, (2016), 3111-3133.

[3] M.G. Grigoryan, L.N. Galoyan, On the universal functions, Journal of Approximation Theory 225, (2018), 191–208.


## The Usage of Lines in $GC_n$-Sets


H. Hakopian and V. Vardanyan

(Yerevan State University, Armenia)

hakop@ysu.am


We consider a $GC_n$ set $\mathcal{X}$, i.e., an $n$-poised set where the fundamental polynomial of each node is a product of $n$ linear factors. We say that a node uses the line $Ax + By + C = 0$ if $Ax + By + C$ divides the fundamental polynomial of this node. It is a known fact that any used line, i.e., a line which is used by a node, passes through at least 2 nodes and through at most $n + 1$ nodes of $\mathcal{X}$. A line is called $k$-node line if it passes through exactly $k$-nodes of $\mathcal{X}$. An $(n + 1)$-node line is called a maximal line. The well-known conjecture of M. Gasca and J. I. Maeztu [1], or briefly GM conjecture, states that every $GC_n$ set has a maximal line. Untill now GM conjecture has been proved only for the cases $n \leq 5$ (see [2], [3]). It is a simple fact that any maximal line $\lambda$ in $\mathcal{X}$ is used by exactly $\binom{n+1}{2}$ nodes, or more precisely, by all the nodes from $\mathcal{X} \setminus \lambda$. As it was proved in [4] and [5] any $n$-node line in $\mathcal{X}$ is used either by exactly $\binom{n}{2}$ or by exactly $\binom{n-1}{2}$ nodes, where $n \in \mathbb{N} \setminus \{3\}$.

Here we adjust and prove a conjecture, proposed in [4], concerning the usage of any $k$-node line in $\mathcal{X}$. Namely, by assuming that GM conjecture is true, we prove that for any $GC_n$-set $\mathcal{X}$ and any $k$-node line $\ell$ the following statements hold:

1. The line $\ell$ is not used at all, or it is used by exactly $\binom{s}{2}$ nodes of $\mathcal{X}$, where $s$ satisfies the condition $\sigma := 2k - n - 1 \leq s \leq k$.

2. If in addition $\sigma \geq 3$ and $\mu(\mathcal{X}) > 3$ then the line $\ell$ is necessarily a used line.

Here $\mu(\mathcal{X})$ denotes the number of maximal lines that the set $\mathcal{X}$ possesses. Moreover we prove that the subset of the nodes from $\mathcal{X}$ that use the line $\ell$ is a $GC_{s-2}$ set with the abovementioned $s$.

At the end, for each $n$ and $k$ with $\sigma = 2$ we bring an example of a $GC_n$-set and a nonused $k$-node line.

# References


[1] M. Gasca, J. I. Maeztu, On Lagrange and Hermite interpolation in $\mathbb{R}^n$, Numer. Math. **39** (1982), 114.

[2] J. R. Busch, A note on Lagrange interpolation in $\mathbb{R}^2$, Rev. Un. Mat. Argentina **36** (1990) 33–38.

[3] H. Hakopian, K. Jetter and G. Zimmermann, The Gasca-Maeztu conjecture for $n = 5$, Numer. Math. **127** (2014) 685–713.

[4] V. Bayramyan, H. Hakopian, On a new property of n-poised and $GC_n$ sets, Adv Comput Math, **43**, (2017) 607-626.

[5] H. Hakopian, V. Vardanyan, On a correction of a property of $GC_n$ sets, Adv Comput Math (2018). https://doi.org/10.1007/s10444-018-9618-4.

## Uniqueness Theorems in Inverse Sturm-Liouville Problems

T. Harutyunyan
(Yerevan State University, Armenia)
hartigr@yahoo.co.uk

In inverse Sturm-Liouville problems, uniqueness theorems of Ambarzumyan, Borg, Marchenko, McLaughlin-Rundell and some others are considered as properties of an analytic function, which we introduced earlier and which we call "Eigenvalues' function of the family of Sturm-Liouville operators".

## A Boundary Value Problem With an Infinite Index

H. Hayrapetyan
(Yerevan State University, Armenia)
hhayrapet@gmail.com

We consider the following Riemann boundary value problem in the unit circle $D^+ = \{z; |z| < 1\}$: define an analytical function $\varphi(z), \varphi(\infty) = 0$ in $D^+ \cup D^-,\ D^- = \{z; |z| < 1\}$ satisfying the condition

$$\lim_{r \to 1-0} \|\varphi^+(rt) - a(t)\varphi^-(r^{-1}t) - f(t)\|_{L^p(\rho)} = 0, \tag{1}$$

where $1 \leq p < \infty$, $\rho(t) = \prod_{k=1}^{\infty} |t_k - t|^{\delta_k}$, $1 > \delta_k > 0$, $\sum_{k=1}^{\infty} \delta_k < \infty$, $t_k \in T$, $T = \{z; |z| = 1\}$. We assume that $t_k = e^{i\theta_k}, \theta_k \downarrow 0$.
For $p = 1$ it is established, that if $\sum_{k=1}^{\infty} \delta_k \ln |1 - t_k| > -\infty$, then the homogeneous problem (1) has an infinite number of linearly independent solutions and the normal solvability of this problem is proved.

# References


[1] **Hayrapetian H.M.**, Dirichlet problem in the half-plane for RO-varying weight functions. Topics in Analysis and its Aplications., *Dordrecht /Boston/ London. Kluwer Academic Publishers.* Vol-147 (2004), 311-317.


## Translation operator on the half-line


A. Horváth

(Budapest University of Technology and Economics, Hungary)

ahorvath@math.bme.hu


Translation operator arised in connection with trigonometric expansions that is translation at the exponent. Generalized translations have been introduced by Delsarte. The extension of the notion and a rather general concept is described in a series of papers by Levitan. During this long period several authors examined and applied the translation operator. As an application, for instance, convolution structures can be defined and Nikol'skii-type inequalities can be derived.

In this talk we investigate translation operator on the half-line. Some old results with new proofs, and some new results will be presented. Some applications will be given as well.

## On Estimation of Functions Orthogonal to Piecewise Constant Functions by the Second Modulus of Continuity

L. Ikhsanov
(Saint Petersburg State University, Russia)
lv.ikhs@gmail.com

The article is concerned with the question about the constant

$$W_2^* = \sup_{f \in F^0} \frac{\|f\|}{\omega_2(f, 1)},$$

there $F^0$ stands for the space of bounded functions $f$ with the property

$$\int_k^{k+1} f(x)\, dx = 0 \quad \forall k \in \mathbb{Z}.$$

The suggested approach allowed to narrow down the known range for the desired constant as well as the set of functions it could be obtained on.

It is shown that $W_2^*$ also happens to be the exact constant in a related Jackson–Stechkin type inequality.

## The Falconer distance problem

A. Iosevich
(University of Rochester, USA)
iosevich@math.rochester.edu

We are going to describe some recent developments pertaining to the Falconer distance problem and its applications to problems in frame theory, combinatorics and beyond.

# On the Sparse Domination for a General Class of Operators

G. Karagulyan

(Insitute of Mathematics, NAS of Armenia)

g.karagulyan@gmail.com

Sparse operators are very simple positive operators recently appeared in the study of weighted estimates of Calderón-Zygmund and other related operators. It was proved that some well-known operators (Calderón-Zygmund operators, martingale transforms, maximal function, Carleson operators, etc.) can be dominated by sparse operators, and this kind of dominations derive series of deep results for the mentioned operators. In particular, Lerner's [6] norm domination of the Calderón-Zygmund operators by sparse operators gave a simple alternative proof to the $A_2$-conjecture solved by Hytönen [3]. Lacey [5] established a pointwise sparse domination for the Calderón-Zygmund operators with an optimal condition (Dini condition) on the modulus of continuity, getting a logarithmic gain to the result previously proved by Conde-Alonso and Rey [1]. The paper [5] also proves a pointwise sparse domination for the martingale transforms, providing a short approach to the $A_2$-theorem proved by Treil-Thiele-Volberg [8]. For the Carleson operators the norms sparse domination was proved by Di Plinio and Lerner [2], while the pointwise domination follows from a general result proved in [4].

The paper [4] introduces a class of so called BO (bounded oscillation) operators on abstract measure spaces, unifying the Calderón-Zygmund operators and maximal functions on general homogeneous spaces, martingale transforms (non-homogeneous case) and Carleson operators. It proves that those operators have pointwise domination by sparse operators. To define BO operators, paper [4] introduces a concept of ball-basis in abstract measure spaces, which is a family of measurable sets holding some common properties of $d$-dimensional balls in $\mathbb{R}^d$ and their analogues in related theories (martingales, dyadic analysis).

# References


[1] José M. Conde-Alonso and Guillermo Rey, *A pointwise estimate for positive dyadic shifts and some applications*, Math. Ann. **365** (2016), no. 3-4, 1111–1135. MR3521084

[2] Francesco Di Plinio and Andrei K. Lerner, *On weighted norm inequalities for the Carleson and Walsh-Carleson operator*, J. Lond. Math. Soc. (2) **90** (2014), no. 3, 654–674. MR3291794

[3] Tuomas P. Hytönen, *The sharp weighted bound for general Calderón-Zygmund operators*, Ann. of Math. (2) **175** (2012), no. 3, 1473–1506. MR2912709

[4] Grigori A. Karagulyan, *An abstract theory of singular operators*, ArXiv e-prints (November 2016), available at `1611.03808`.

[5] Michael T. Lacey, *An elementary proof of the $A_2$ bound*, Israel J. Math. **217** (2017), no. 1, 181–195. MR3625108

[6] Andrei K. Lerner, *On an estimate of Calderón-Zygmund operators by dyadic positive operators*, J. Anal. Math. **121** (2013), 141–161. MR3127380

[7] ———, *A simple proof of the $A_2$ conjecture*, Int. Math. Res. Not. IMRN **14** (2013), 3159–3170. MR3085756

[8] Christoph Thiele, Sergei Treil, and Alexander Volberg, *Weighted martingale multipliers in the non-homogeneous setting and outer measure spaces*, Adv. Math. **285** (2015), 1155–1188. MR3406523


## An exponential estimate for the cubic partial sums of multiple Fourier series


G. Karagulyan and H. Mkoyan

(Yerevan State University, Armenia)

g.karagulyan@ysu.am, hasmikmkoyan@ysu.am


Let $\mathbb{T} = \mathbb{R}/2\pi$ and $\mathbb{T}^d$ denote the $d$-dimensional torus. The multiple trigonometric Fourier series of a function $f \in L^1(\mathbb{T}^d)$ and its conjugate are the series

$$\sum_{\mathbf{n}=(n_1,\dots,n_d)\in\mathbb{Z}^d} a_{\mathbf{n}} e^{i\mathbf{n}\cdot\mathbf{x}}, \tag{1}$$

$$\sum_{\mathbf{n}=(n_1,\dots,n_d)\in\mathbb{Z}^d} \left(\prod_{k=1}^{d}(-i\cdot\operatorname{sign} n_k)\right) a_{\mathbf{n}} e^{i\mathbf{n}\cdot\mathbf{x}}, \tag{2}$$

where

$$\mathbf{n}=(n_1,\dots,n_d), \quad \mathbf{x}=(x_1,\dots,x_d),$$
$$\mathbf{n}\cdot\mathbf{x}=n_1x_1+\dots+n_dx_d,$$
$$a_{\mathbf{n}}=\frac{1}{(2\pi)^d}\int_{\mathbb{T}^d} f(\mathbf{x})\, e^{-i\mathbf{n}\cdot\mathbf{x}}d\mathbf{x}.$$

Denote the cubic partial sums of series (1) by

$$S_n f(\mathbf{x})=\sum_{-n\le k_i\le n} a_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}, \quad n\in\mathbb{N},$$

and let $\tilde{S}_n$ be its conjugate.

Consider the Orlicz classes of functions corresponding to the logarithmic functions

$$\operatorname{Log}_k(u)=|u|\max\{0,\log^k|u|\}, \quad k=1,2,\dots,$$

that is, the Banach space of functions

$$\operatorname{Log}_k(L)(\mathbb{T}^d)=\left\{f\in L^1(\mathbb{T}^d):\int_{\mathbb{T}^d}\operatorname{Log}_k(f)<\infty\right\}$$

with the Luxemburg norm

$$\|f\|_{\operatorname{Log}_k(L)}=\inf\left\{\lambda:\lambda>0,\int\limits_{\mathbb{T}^d}\operatorname{Log}_k\left(\frac{f}{\lambda}\right)\le 1\right\}<\infty.$$

The main result of the paper is the following:

**Theorem 1.** *For any* $f \in \operatorname{Log}_{d-1}(L)(\mathbb{T}^d)$ *there exists a measurable function* $F(\boldsymbol{x}) > 0$ *on* $\mathbb{T}^d$ *such that*

$$|\{\boldsymbol{x} \in \mathbb{T}^d : F(\boldsymbol{x}) > \lambda\}| \lesssim \frac{\|f\|_{\operatorname{Log}_{d-1}(\mathbb{T}^d)}}{\lambda},$$

$$\int_{\mathbb{T}^d} \exp\left(\frac{|S_n f(\mathbf{x})| + |\tilde{S}_n f(\mathbf{x})|}{F(\boldsymbol{x})}\right) d\mathbf{x} \lesssim 1, \quad n = 1, 2, \ldots .$$

The relation $a \lesssim b$ in the theorem stands for the inequality $a \leq c \cdot b$, where $c$ is a constant that can depend only on the dimension $d$.

## Boundary Embedding Theorems for Multianisotropic Spaces

G. Karapetyan
(Russian-Armenian University, Armenia)
garnik_karapetyan@yahoo.com

In the proof of all embedding theorems (in particular, see [1]), two cases are distinguished. The first case is when the embedding index is less than one; the second case is when the exponent is one, that is the boundary case holds. In previous papers we studied embedding theorems for functions from multianisotropic spaces (see [2] - [3]), when the embedding index is less than unity. In this paper we prove embedding theorems for multianisotropic function spaces in the boundary case.

For any parameter $\nu > 0$ and a natural number $k$ denote

$$P(\nu, \xi) = \left(\nu \xi^{\alpha^1}\right)^{2k} + \cdots + \left(\nu \xi^{\alpha^n}\right)^{2k} + \left(\nu \xi^{\alpha^{n+1}}\right)^{2k}.$$

$$G_0(\nu; \xi) = e^{-P(\nu, \xi)}.$$

$$G_{1,j}(\nu,\xi) = 2k\left(\nu\xi^{\alpha^j}\right)^{2k-1} e^{-P(\nu,\xi)}, (j = 1,\ldots,n+1).$$

For any function $f$ consider the regularization with the kernel $\hat{G}_0(t,\nu)$:

$$f_\nu(x) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int\limits_{R^n} f(t)\hat{G}_0(t-x,\nu)dt.$$

The following integral representation holds:

**Theorem 1.** *Let the function $f$ have the Sobolev weak derivatives $D^{\alpha^i} f$, $(i = 1,\ldots,n+1)$, where $\alpha^i$ are the vertices of the completely regular polyhedron $\mathfrak{N}$ and $D^{\alpha^i} f \in L_p(R^n)$, $1 \leq p < \infty$, $(i = 1,\ldots,n+1)$. Then for almost all $x \in R^n$ it has the representation*

$$f(x) = f_h(x) + \lim_{\varepsilon\to 0} \sum_{i=1}^{n+1} \frac{1}{(2\pi)^{\frac{n}{2}}} \int\limits_{\varepsilon}^{h} d\nu \int\limits_{R^n} D^{\alpha^i} f(t)\hat{G}_{1,i}(t-x,\nu)dt.$$

Let $\mathfrak{N}$ be a completely regular polyhedron, then $W_p^{\mathfrak{N}}(\mathbb{R}^n) = \{f : f \in L_p(\mathbb{R}^n), D^{\alpha^i} f \in L_p(\mathbb{R}^n), i = 1,\ldots,M\}$.

The main result of this paper is the following boundary embedding theorem for functions from a multianisotropic space $W_p^{\mathfrak{N}}(\mathbb{R}^n)$ $(p > 1)$:

**Theorem 2.** *Let the numbers $p$ and $q$ satisfy the relations $1 < p \leq q < \infty$ and a multi-index $\beta = (\beta_1,\ldots,\beta_n)$ such that*

$$\chi = \max_{i=1,\ldots,I_{n-1}} \left(\left(\beta,\mu^i\right) + \left|\mu^i\right| \left(\frac{1}{p} - \frac{1}{q}\right)\right) = 1.$$

*Then $D^\beta W_p^{\mathfrak{N}}(\mathbb{R}^n) \hookrightarrow L_q(\mathbb{R}^n)$, i.e. any function $f \in W_p^{\mathfrak{N}}(\mathbb{R}^n)$ has weak derivatives $D^\beta f$, belonging to the class $L_q(\mathbb{R}^n)$, and for some constants $C_1, C_2 > 0$ the following inequality holds*

$$\left\|D^\beta f\right\|_{L_q(\mathbb{R}^n)} \leq C_1 \sum_{i=1}^{M} \left\|D^{\alpha^i} f\right\|_{L_p(\mathbb{R}^n)} + C_2\|f\|_{L_p(\mathbb{R}^n)}.$$

# References

[1] *Besov O.V., Il'in V.P., Nikloskii S.M.* Integral representations of functions and embedding theorems. Nauka, Moscow, 1975 (in Russian), p. 480.

[2] *Karapetyan G. A.* Integral representation of functions and embedding theorems for multianisotropic spaces on a plane. Journal of Contemporary Mathematical Analysis, 2016 (in press).

[3] *Karapetyan G. A., Arakelian M. K.* Embedding theorems for general multianisotropic spaces. Mathematical notes (in press).

## Construction of Approximate Solutions of the Dirichlet Problem in $\mathbb{R}^n$ for Regular Hypoelliptic Operators

G. Karapetyan and H. Petrosyan

(Russian-Armenian University, Armenia)

garnik_karapetyan@yahoo.com, heghine.petrosyan@rau.am

In this paper we consider the Dirichlet problem in a half-space for special (multihomogeneous) regular hypoelliptic equations with zero boundary conditions. Problems of this type appear in the study of multianisotropic processes and the difficulty of their study lies in the fact that the corresponding characteristic polynomial is not generalized homogeneous, as for elliptic or semi-elliptic equations (see [1]), but a multihomogeneous one, and the construction of an approximate solution for such equations is difficult by the nature. But, applying special integral representations of functions (see [2]) through vertices of a completely regular Newton polyhedron, it was possible to construct approximate solutions in terms of integral operators. Similar questions in the whole space $\mathbb{R}^n$ were studied in [3]. In this paper we study the question of the solvability of the Dirichlet problem in Sobolev spaces $W_p^{\mathfrak{N}}(\mathbb{R}^n_+)$ $(1 < p < \infty)$.

Consider the differential operator $P(D_x, D_{x_n})$ in $\mathbb{R}^n_+$ with constant real coefficients $a_i$ $(i = 1, \dots, M)$

$$P(D_x, D_{x_n}) = D^{2m}_{x_n} + \sum_{i=1}^{M} a_i D^{\alpha^i}. \tag{1}$$

Assume that the operator (1) is a regular operator, i.e. there exists a constant number $C > 0$ such that for any $\xi \in \mathbb{R}^n$ the following inequality holds:

$$|P(\xi, \xi_n)| \geq C \left( \sum_{i=1}^{M} \left| \xi^{\alpha^i} \right| + \xi_n^{2m} \right). \tag{2}$$

We also denote by $\chi = \min\limits_{i=1,\dots,I_{n-2}} \left( |\mu^i| + \frac{1}{2m} \right) - \left( |\mu^1| + \frac{1}{2m} \right) \frac{1}{p}$, where $p > 1$ is some number.

In $\mathbb{R}^n_+$ consider the following Dirichlet problem:

$$\begin{cases} P(D_x, D_{x_n})U = f(x, x_n), \quad x_n > 0,\ x \in \mathbb{R}^{n-1}, & (3) \\ \left. \dfrac{\partial^i U}{\partial x_n^i} \right|_{x_n=0} = 0, \quad i = 0, 1, \dots, m-1. & (4) \end{cases}$$

In this paper we study the solvability of the problem (3)-(4). Namely, we prove the following theorems:

**Theorem 1.** *If $f \in L_p(\mathbb{R}^n_+)$ $(1 < p < \infty)$ and has a compact support, then for $\chi > 1$ the problem (3)-(4) has a unique solution $U$ from the class $W^{\mathfrak{M}}_p(\mathbb{R}^n_+)$, and for some constant $C > 0$ (independent of $f$) we have the estimate*

$$\|U\|_{W^{\mathfrak{M}}_p(\mathbb{R}^n_+)} \leq C \|f\|_{L_p(\mathbb{R}^n_+)}. \tag{5}$$

And when $\chi \leq 1$ the following theorem holds.

**Theorem 2.** *Let $\chi \leq 1$ and $f \in L_p(\mathbb{R}^n_+)$ $(1 < p < \infty)$ with compact support satisfies the following orthogonality conditions:* $\int\limits_{\mathbb{R}^{n-1}} x^s f(x, x_n) dx = 0$

*as* $|s| = 0, 1, \ldots, N-1$, *where* $N$ *is a natural number, determined from the inequalities* $\chi + N\mu^{i_0}_{min} > 1 > \chi + (N-1)\mu^{i_0}_{min}$, *where* $\mu^{i_0}_{min} = \min\limits_{j=1,\ldots,n} \mu^{i_0}_j$, *and* $i_0$ *is the index for which* $\min\limits_{i=1,\ldots,I_{n-2}} |\mu^i| = |\mu^{i_0}|$. *Then for any such function* $f$ *the problem (3)-(4) has a unique solution from the class* $W^{\mathfrak{m}}_p(\mathbb{R}^n_+)$, *for which the inequality (5) holds.*

# References


[1] *Demidenko G. V.* On the correct solvability of boundary value problems in a half-space for quasielliptic equations // Siberian Mathematical Journal, Vol. XXIX, No. 4, (1988).

[2] *Karapetyan G. A.* Integral representation of functions and embedding theorems for $n$-dimensional multianisotropic spaces with one vertex of anisotropicity. // Siberian Math. Journal, v.58, n.3, (2017), 445-460.

[3] *Karapetyan G. A., Petrosyan H. A.* On the solvability of regular hypoelliptic equations in $R^n$ // Journal of Contemporary Mathematical Analysis, (in press).


## On Grand and Small Ap Spaces


A. Karapetyants

(Southern Federal University, Don State Technical University, Russia)

karapetyants@gmail.com



We introduce grand and small Ap spaces of functions holomorphic in the unit disc. We prove the boundedness of the projection operator in such grand Ap space. As the main result we prove estimates for functions in grand or small Ap spaces near the boundary which differ from the case of the classical Ap space by logarithmic factor with positive or negative exponent respectively.

# Subdivision schemes on a dyadic half-line

M. Karapetyants
(Moscow Institute of Physics and Technology, Russia)
karapetyantsmk@gmail.com

We consider the subdivision operator on a dyadic half-line. Necessary and sufficient convergence conditions, the connection between the subdivision scheme and the refinement equation, existence criterion of a fractal curve and continuous solution of the refinement equation and some combinatorial properties of a subdivision scheme are studied. The conjecture on convergence of subdivision schemes with non-negative masks is also formulated.

## References

[1] *Protasov V. Yu.* Dyadic wavelets and refinable functions on a half-line. Sbornik: Mathematics. 2006. Vol. 197, No. 10, pp. 129–160.

[2] *Golubov B. I., Efimov A. V., Skvortsov V. A.* Walsh series and transforms: theory and applications. Nauka.1987.

[3] *Daubechies I.* Ten lectures on wavelets. SIAM.1992.

[4] *Golubov B. I.* Binary analysis elements. LKI.2007.

[5] *Novikov I. A., Protasov V. Yu., Skopina M. A.* Wavelet theory. PhysMatLit.2005.

[6] *Melkman A. A.* Subdivision schemes with non-negative masks converge always - unless they obviously cannot. Baltzer Journals. 1996.

# On Continuity of the Best Approximations by Constants

I. Katkovskaya and V. Krotov
(Belarusian National Technical University, Belarusian State University, Belarus)
ikatkovskaya@bntu.by, vgkrotov@mail.ru

Let $(X, d, \mu)$ be a bounded metric space with the metric $d$ and Borel measure $\mu$. Denote by

$$B(x,r) = \{y \in X : d(x,y) < r\}, \quad C(x,r) = \{y \in X : d(x,y) = r\}$$

the ball and the sphere with the center at $x \in X$ and radius $r > 0$ respectively.

If $p > 0$ and $f \in L^p(X)$, then for any ball $B \subset X$ there exists a number $I_B^{(p)} f \in \mathbb{R}$ such that

$$\inf_{I \in \mathbb{R}} \int_B |f(y) - I|^p \, d\mu(y) = \int_B |f(y) - I_B^{(p)} f|^p \, d\mu(y).$$

For $p > 1$ such number $I_B^{(p)} f$ is unique. If $0 < p \leq 1$ then the constant $I_B^{(p)} f$ can be non-unique. Denote by $\mathcal{BA}_p(f, B)$ the set of all such constants, and

$$\mathbb{I}_B^{(p)} f = \inf \mathcal{BA}_p(f, B), \quad \mathbf{I}_B^{(p)} f = \sup \mathcal{BA}_p(f, B).$$

From general theorems of multivalued analysis it is not difficult to deduce the measurability of these extremal best approximations, as functions of the center of the ball.

**Theorem 1.** *If $p > 0$, then for any ball $B \subset X$ 1) the function $f \mapsto \mathbb{I}_B^{(p)} f$, $f \in L^p(B)$, is lower semicontinuous on $L^p(B)$; 2) the function $f \mapsto \mathbf{I}_B^{(p)} f$, $f \in L^p(B)$, is upper semicontinuous on $L^p(B)$.*

In particular, if $f_0 \in L^p(B)$, $p > 0$ and the set $\mathcal{BA}_p(f_0, B)$ consists of only one element, then the function $f \mapsto \mathbb{I}_B^{(p)} f$, $f \in L^p(B)$ is continuous at $f_0$.

In the following theorem, we assume that there exists a constant $a_\mu \geq 1$ such that

$$\mu(B(x, 2r)) \leq a_\mu \mu(B(x, r)), \quad x \in X, \ r > 0$$

(the doubling condition).

**Theorem 2.** *Let* $p > 0$, $x_0 \in X$, $r > 0$ *and*

$$\mu(C(x_0, r)) = 0. \tag{1}$$

*Then 1) the function* $x \mapsto \mathbb{I}_{B(x,r)}^{(p)} f$, $x \in X$, *is lower semicontinuous at the point* $x_0$*; 2) the function* $x \mapsto \mathbf{I}_{B(x,r)}^{(p)} f$, $x \in X$ *is upper semicontinuous at the point* $x_0$.

In particular, if $f \in L^p(X)$, $p > 0$, $x_0 \in X$, $r_0 > 0$ and the condition (1) is satisfied, then if the set $\mathcal{BA}_p(f, B(x_0, r_0))$consists of only one element, then the function $x \mapsto \mathbb{I}_{B(x,r_0)}^{(p)} f$, $x \in X$, is continuous at the point $x_0$.

We note that the condition (1) in Theorem 2 is essential. Without it the assertion of Theorem 2 is not true.

# On the Relation of Stability Between Linear Ordinary Systems, Impulsive and Difference Equations and Linear Systems of Generalized Ordinary Differential Equations

N. Kekelia
( Sokhumi State University, Georgia)
nestanikekelia@yahoo.com

We consider a linear homogeneous system: of ordinary differential equations

$$\frac{dx}{dt} = p(t),$$

impulsive equations

$$\frac{dx}{dt} = Q(t),$$

$$x(t_j+) - x(t_j-) = G_{(j)} x(t_j-) (j = 1, 2, ...)$$

and of difference equations

$$\Delta y(k-1) = G(k-1) y(k-1) (k \in N).$$

We give some sufficient (among them effective) conditions. If these conditions are fulfilled, then the stability in Liapunov sense of the above-mentioned systems implies that of the linear homogeneous system of the generalized ordinary differential equations

$$dx(t) = dA(t) \bullet x(t)$$

in the same sense.

Here we assume that $P, Q : R_+ \to R^{(n\times n)}$( $R_+ = [0; +\infty[$ are matrix-functions with Lebesgue integrable components on every closed interval from $R_+$; $G_j \in R^{(n\times n)}$ $(j = 1, 2, \ldots)$ are constant matrices, $t_j \in R_+ (j = 1, 2, \ldots), 0 < t_1 < t_2 < \cdots, \lim_{j\to\infty} t_j = +\infty, G : \{0, 1, \ldots\} \to R^{(n\times n)}$ is a matrix-function with bounded total variation components on every closed interval from $R_+$).

To a considerable extent, the interest to the theory of generalized ordinary differential equations has also been stimulated by the fact that this theory enables one to investigate ordinary differential, impulsive and difference equations from a unified point of view (see [1]-[3]).

# References


[1] S. Schwabik, M. Tvrdy, and O. Vejvoda, Differential and integral equations. Boundary value problems and adjoints. D. Reidel Publishing Co., Dordrecht-Boston, Mass.-London, 1979.

[2] M. Ashordia and N. Kekelia, On the $\zeta$-exponentially asymptotic stability of linear systems of generalized ordinary differential equations. Georgian Math. J. 8(2001), No. 4, 645-664.

[3] M. Ashordia, On the general and multipoint boundary value problems for linear systems of generalized ordinary differential equations, linear impulse and linear difference systems. Mem. Differential Equations Math. Phys. 36(2005), 1- 80.

# Building Blocks for Function Spaces With Variable Exponents

H. Kempka
(University of Applied Sciences, Jena, Germany)
henning.kempka@eah-jena.de

To describe function spaces of Besov and Triebel-Lizorkin type by different building blocks as atoms, molecules and wavelets are used to decompose the functions. Usually, these building blocks have different properties as compact support or decay conditions and the characterization of the corresponding function space relies on a special interplay of smoothness and moment conditions.

It was already observed by Frazier and Jawerth [1] in 1985 that atoms can be used as building blocks to characterize Besov spaces $B^s_{p,q}$. This result could be generalized in [2] for atoms, molecules and wavelets as building blocks to characterize variable Besov $B^{s(\cdot)}_{p(\cdot),q(\cdot)}$ and Triebel-Lizorkin spaces $F^{s(\cdot)}_{p(\cdot),q(\cdot)}$ where the smoothness $s(\cdot)$ as well as the integrability parameters $p(\cdot)$ and $q(\cdot)$ depend on the space variable $x \in \mathbb{R}^n$.

In this talk we will discuss these concepts and also present the recent result from [3] where even non-smooth atoms are used to characterize these variable function spaces.

# References

[1] M. Frazier and B. Jawerth, *Decomposition of Besov spaces.* Indiana Univ. Math. J. **34** (1985), 777–799.

[2] H. Kempka, *Atomic, molecular and wavelet decomposition of 2-microlocal Besov and Triebel-Lizorkin spaces with variable integrability.* Funct. Approx. Comment. Math., **43**, no. 2 (2010), 171–208.

[3] H. Kempka and H. Gonçalves, *Non-smooth atomic decomposition of 2-microlocal spaces and application to pointwise multipliers.* J. Math. Anal. Appl. **434** (2016), 1875–1890.

# Regularity Spaces, and Wavelet in a Geometric Framework. Application to Gaussian Processes and Statistical Estimation

G. Kerkyacharian

(Université Pierre et Marie Curie, France)

kerk@math.jussieu.fr

We will show how, on a suitable Dirichlet space, one can define regularity spaces, and a wavelet system. This construction works for Riemannian manifolds, under some curvature condition. Then we revisit some classical results on the behavior of Gaussian processes, and non parametric adaptive statistical estimation.

## References

[1] T. Coulhon, G. Kerkyacharian, and P. Petrushev, Heat Kernel Generated Frames in the Setting of Dirichlet Spaces, J. Fourier Anal. Appl. 18 (2012), no. 5, 995–1066.

[2] G. Kerkyacharian, P. Petrushev, Heat kernel based decomposition of spaces of distributions in the framework of Dirichlet spaces, Trans. Amer. Math. Soc. 367 (2015), no. 1, 121–189.

[3] G. Kerkyacharian, S. Ogawa, P. Petrushev, D. Picard, Regularity of Gaussian processes on Dirichlet spaces, Constr. Approx. 47 (2018), no. 2, 277–320.

# Projection Operators Onto Spaces of Chebyshev Splines

K. Keryan and M. Passenbrunner
(Yerevan State University, Armenia; Johannes Kepler University Linz, Austria)
karenkeryan@ysu.am, markus.passenbrunner@jku.at

We extend Shadrin's theorem on the boundedness of the polynomial spline orthoprojector on $L^\infty$ by a constant that does not depend on the underlying univariate grid $\Delta$ to the setting of Chebyshev splines. The space of Chebyshev splines $S(\mathcal{U}_k; \Delta)$ of order $k$ consists of functions that, on each grid interval of $\Delta$, are contained in $\mathcal{U}_k = \operatorname{span}\{u_1, \ldots, u_k\}$, the space of linear combinations of generalized monomials $u_1, \ldots, u_k$ that arise by iterated integrals of positive weight functions in the same way as the classical monomials $1, \ldots, x^{k-1}$ arise as iterated integrals of constant functions.

The main result is the following

**Theorem 1.** *There exists a constant $C$, depending only on $\mathcal{U}_k$, so that for all partitions $\Delta$ of the unit interval $[0,1]$, the orthogonal projection operator $P_\Delta$ onto the space of Chebyshev splines $S(\mathcal{U}_k; \Delta)$ is bounded on $L^\infty$ by the constant $C$, i.e.,*

$$\|P_\Delta f\|_\infty \leq C\|f\|_\infty, \qquad f \in L^\infty[0,1].$$

# Gradient Descent Method in Parametric Optimization Problems

R. Khachatryan
(Yerevan State University, Armenia)
khachatryan.rafik@gmail.com

Given a set $M \subseteq R^m$ of strategies of the first player, the set of $\varepsilon$-optimal strategies of the second player with loss function $f(x, y)$ is defined by

$$a_\varepsilon(x) = \{y \in M / f(x,y) \leq \inf_{y \in M} f(x,y) + \varepsilon\}.$$

Using the gradient descent method we construct a functional sequence and prove that under certain conditions the distance of this sequence from the set $a_\varepsilon(x)$ tends to zero uniformly on $E$, where $E \subset R^n$ is a convex compact.

**Theorem**. Let $M = R^m$ and $f(x, y)$ by a continuous function satisfying the following conditions:

1) $f(x, y)$ is convex with respect to $y$ for fixed $x \in E$, where $E$ is some neighborhood of $x_0$;

2) for each $x \in E$ the partial derivative $f'_y(x,y) = \frac{\partial f(x,y)}{\partial y}$ exists and is continuous with respect to $x$ and $y$;

3) the set $a_0(x_0) = \{y \in R^m / f(x_0, y) = inf_{v \in R^m} f(x_0, v)\}$ is nonempty and bounded.

Let

$$\lambda_j \downarrow 0, \ \sum_{j=0}^{\infty} \lambda_j = \infty, \ \sum_{j=0}^{\infty} \lambda_j^2 < \infty.$$

Then there exists a closed neighborhood $E_0 \subseteq E$ of $x_0$ such that for any $\varepsilon > 0$

$$\inf_{u \in a_\varepsilon(x)} \| y_j(x) - u \| \to 0$$

as $j \to \infty$ uniformly on $x \in E_0$, where

$$y_{j+1}(x) = y_j(x) - \lambda_j \frac{f'_y(x, y_j(x))}{\max_{x \in E_0} \| f'_y(x, y_j(x)) \|}.$$

## On Steklov Means in Operator Cosine Function Framework in Banach Space

A. Kivinukk and A. Seletski
(Tallinn University, Estonia)
andik@tlu.ee, annat@tlu.ee

The Steklov means are used in approximation theory of functions in different aspects [1],[3],[4]. This talk concerns the Steklov means by using the operator cosine function framework. The operator cosine function [2] offers a counterpart of the translation operator, which forms the basic concept for the modulus of continuity and for some approximation processes as well. For example, the Rogosinski approximation processes are defined as the arithmetical mean of shifted Fourier partial sums. We will show that the operator cosine function concept allows to define very general Steklov means in an abstract Banach space. The approximation properties of these generalized Steklov means appear to be quite similar to the properties of the Steklov means in trigonometric approximation.

# References

[1] Akhiezer, N. I., Lectures in the Theory of Approximation. [in Russian] second revised and enlarged edition, Izdat. "Nauka", Moscow, 1965, 307 pp.

[2] Lutz D., Strongly continuous operator cosine functions, In: Functional Analysis. Proc., Dubrovnik 1981, Lect Notes in Math. 948, 1982, Eds. Butković,D. , Kaljević, H., Kurepa, S.

[3] Vinogradov, O. L., Zhuk, V. V., Estimates for functionals with a known moment sequence in terms of deviations of Steklov type means. J Math Sci, Springer US, 2011, 178:115 (Translated from *Zapiski Nauchnykh Seminarov POMI*, Vol. 383, 2010, pp. 5–32.)

[4] Zhuk, V. V., Inequalities of the type of the generalized Jackson theorem for the best approximations. J Math Sci, Springer US, 2013, Vol. 193, Issue 1, pp 75–88 (Translated from *Zapiski Nauchnykh Seminarov POMI*, Vol. 404, 2012, pp. 135–156.)

# The Lower Bound for the Volume of the Image of a Ball

B. Klishchuk and R. Salimov

(Institute of Mathematics of NAS, Ukraine)

kban1988@gmail.com

Assume that we are given a family $\Gamma$ of curves $\gamma$ in the space $\mathbb{R}^n$, $n \geqslant 2$. A Borel function $\rho : \mathbb{R}^n \to [0, \infty]$ is called *admissible* for $\Gamma$, which is written as $\rho \in \operatorname{adm}\Gamma$, if

$$\int\limits_{\gamma} \rho(x)\, ds \ \geqslant\ 1$$

for an arbitrary curve $\gamma \in \Gamma$.

Let $p \in (1, \infty)$. Then a *p–modulus* of the family $\Gamma$ is the quantity

$$\mathcal{M}_p(\Gamma) \ =\ \inf_{\rho \in \operatorname{adm}\Gamma} \int\limits_{\mathbb{R}^n} \rho^p(x)\, dm(x)\,.$$

Here $m$ is the Lebesgue measure in $\mathbb{R}^n$.

For arbitrary sets $E$, $F$ and $G$ in $\mathbb{R}^n$, denote by $\Delta(E, F, G)$ a family of all continuous curves $\gamma : [a, b] \to \mathbb{R}^n$, connecting $E$ and $F$ in $G$, i.e. $\gamma(a) \in E$, $\gamma(b) \in F$ and $\gamma(t) \in G$ for $a < t < b$.

Assume that $D$ is a domain in the space $\mathbb{R}^n$, $n \geqslant 2$, $x_0 \in D$ and $d_0 = \operatorname{dist}(x_0, \partial D)$. Let

$$\mathbb{A}(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x - x_0| < r_2\}\,,$$

$$S_i = S(x_0, r_i) = \{x \in \mathbb{R}^n :\ |x - x_0| = r_i\}\,, \quad i = 1, 2\,.$$

Let $Q : D \to [0, \infty]$ be a Lebesgue measurable function. A homeomorphism $f : D \to \mathbb{R}^n$ is called a ring $Q$-homeomorphism with respect to $p$-modulus at the point $x_0 \in D$, if the esimate

$$\mathcal{M}_p(\Delta(fS_1, fS_2, fD)) \ \leqslant \int\limits_{\mathbb{A}} Q(x)\, \eta^p(|x - x_0|)\, dm(x)$$

holds true for any ring $\mathbb{A} = \mathbb{A}(x_0, r_1, r_2)$, $0 < r_1 < r_2 < d_0$, and for every measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that

$$\int\limits_{r_1}^{r_2} \eta(r)\, dr = 1\,.$$

**Theorem 1.** *Let $D$ be a bounded domain in $\mathbb{R}^n$, $n \geqslant 2$, and $f : D \to \mathbb{R}^n$ be a ring $Q$-homeomorphism with respect to $p$-modulus at the point $x_0 \in D$ for $p > n$. Assume that the function $Q$ satisfies the condition*

$$q_{x_0}(t) \leqslant q_0\, t^{-\alpha},\ q_0 \in (0, \infty)\,,\ \alpha \in [0, \infty)\,,$$

*for $x_0 \in D$ and almost all $t \in (0, d_0)$, $d_0 = \operatorname{dist}(x_0, \partial D)$. Then for all $r \in (0, d_0)$ the estimate*

$$m(fB(x_0, r)) \geqslant \Omega_n \left(\frac{p - n}{\alpha + p - n}\right)^{\frac{n(p-1)}{p-n}} q_0^{\frac{n}{n-p}}\, r^{\frac{n(\alpha+p-n)}{p-n}}\,,$$

*holds true, where*

$$B(x_0, r) = \{x \in \mathbb{R}^n : |x - x_0| \leqslant r\},\ q_{x_0}(t) = \frac{1}{\omega_{n-1}\, r^{n-1}} \int\limits_{S(x_0, r)} Q(x)\, d\mathcal{A}$$

*is the integral mean value over the sphere* $S(x_0, t) = \{x \in \mathbb{R}^n : |x - x_0| = t\}$, $\Omega_n$ *is the volume of the unit ball in* $\mathbb{R}^n$, $\omega_{n-1}$ *is the surface area of the unit sphere* $\mathbb{S}^{n-1}$ *in* $\mathbb{R}^n$, $d\mathcal{A}$ *is the element of the surface area.*

## On the Convergence of Double Fourier Series With Respect to Haar and Franklin Systems

A. Kobelyan
(Yerevan State University, Armenia)
a_kobelyan@ysu.am

The main aim of the present talk is to present the following theorems for Franklin and Haar systems:

**Theorem A.** For any $0 < \varepsilon < 1$ there exists a measurable set $E \subset [0,1]^2$ with measure $|E| > 1 - \varepsilon$ such that, for any function $f(t,\tau) \in L^1[0,1)^2$, there exists a function $\tilde{f}(t,\tau) \in L^1[0,1)^2$ which coincides with $f(t,\tau)$ on $E$ and such that its Fourier series with respect to the double Franklin system converges to it in the sense of both spherical and rectangular partial sums almost everywhere on $[0,1)^2$.

**Theorem B.** For any $0 < \varepsilon < 1$ there exists a measurable set $E \subset [0,1]^2$ with measure $|E| > 1 - \varepsilon$ such that, for any function $f(t,\tau) \in L^1[0,1)^2$, there exists a function $\tilde{f}(t,\tau) \in L^1[0,1)^2$ which coincides with $f(t,\tau)$ on $E$ and such that its Fourier series with respect to the double Haar system converges to it in the sense of both spherical and rectangular partial sums almost everywhere on $[0,1)^2$, furthermore the sequence of nonzero Fourier coefficients of the new function with respect to the double Haar system are arranged in decreasing order in all directions.

Note that from theorem B immediately follows that the spherical common terms of double Fourier-Haar series of modified function $\tilde{f}(t,\tau)$ converges to zero a.e..

Note also the result of Oniani, according to which for each function $f(x,y) \in L(\ln^+ L)$ one can find a function $g(x,y) \geq 0$, which equidistributed with it, and such that the rectangular and spherical common terms of double Fourier-Haar series of $g$ are unbounded a.e. on $[0,1)^2$.

# References


[1] L. N. Galoyan, M. G. Grigoryan, A. Kh. Kobelyan, Convergence of Fourier series in classical systems, Sbornik: Mathematics, 2015, 206:7, 941-979.

[2] A.Kh. Kobelyan, On the absolute convergence Haar series, Advances in Theoretical and Applied Mathematics. V. 12, N. 2 (2017), pp. 81-84.

[3] G. G. Oniani, On the divergence of multiple Fourier-Haar series, Analysis Mathematica, 38(2012), 227-247.


## New Trends in Non-Standard Function Spaces and Integral Operator: Theory With Applications


V. Kokilashvili

(Tbilisi State University, Georgia)

vakhtangkokilashvili@yahoo.com


The main goal of the lecture is to discuss the following topics: new "grand" spaces, extrapolation results, boundedness of sublinear operators, sharp weighted bounds, applications to PDEs.

## Convergence to Zero of Exponential Sums With Positive Integer Coefficients

S. Konyagin
(Steklov Mathematical Institute, Russia)
konyagin23@gmail.com

We prove that there is a sequence of trigonometric polynomials with positive integer coefficients, which converges to zero almost everywhere.

# References

[1] P. A. Borodin, S. V. Konyagin, Convergence to zero of exponential sums with positive integer coefficients and approximation by sums of shifts of a single function on the line, Anal. Math. 44, N. 2, 163–183 (2018).

## Geometry and Convergence of Products of Projections

E. Kopecka
(University of Innsbruck, Austria)
eva.kopecka@uibk.ac.at

We investigate when slow convergence of cyclic products of projections implies divergence of random products of projections and vice versa.

Suppose $L_1, \dots, L_K$ are closed subspaces of a Hilbert space $H$ and $P_{L_i}$ are the orthogonal projections onto them. It is known that the cyclic product $(P_K \dots P_1)^n x$ converges to the projection of $x$ onto $\bigcap L_i$ for each starting point $x \in H$. Moreover, there is a dichotomy: the product converges exponentially fast iff $L_1^{\perp} + \cdots + L_K^{\perp}$ is closed; otherwise it converges as slow as one likes for an appropriately chosen initial vector $x \in H$. Deutsch and Hundal asked if the closedness of $L_1^\perp + \cdots + L_K^\perp$ implies the convergence of $z_n = P_{L_{k_n}} z_{n-1}$ for any starting point $z \in H$ and any sequence $k_1, k_2, \dots \in \{1, \dots, K\}$. We give an example that for $K \geq 5$ this is not the case. However, for $K = 3$ and $K = 4$ we show that $L_1^\perp + \cdots + L_K^\perp$ is closed if and only if $z_n = P_{\tilde{L}_{k_n}} z_{n-1}$ converges for each $z \in H$, all closed subspaces $\tilde{L}_i \subset L_i$ and all sequences $k_1, k_2, \dots \in \{1, \dots, K\}$. We also explain what happens when $k \geq 5$.

## Multivariate Sampling-Type Expansions

A. Krivoshein
(Saint Petersburg State University, Russia)
krivosheinav@gmail.com

Approximation properties of expansions $\sum_k c_k \varphi(M^j \cdot -k)$ are studied, where $M$ is a matrix dilation, $c_k$ are sampled values of $f$, i.e. $f(M^{-j}k)$, or sampled values of an appropriate differential operator $L$, i.e. $Lf(M^{-j}\cdot)(k)$, or the integral averages of $f$ near $M^{-j}k$. Error estimations in $L_p$-norm, $p \geq 2$, are given in terms of the Fourier transform of $f$. The approximation order depends on how smooth is $f$, on the order of Strang-Fix condition for $\varphi$ and on $M$. Some special properties of $\varphi$ are required, but the class of functions $\varphi$ we consider is large enough (including compactly supported splines as well as band-limited functions). Periodic case is also discussed.

This is a joint work with M. Skopina.

# Continuous sums of ridge functions on a convex body

A. Kuleshov
(Moscow State University, Steklov Mathematical Institute, Russia)
kuleshov.a.a@yandex.ru

Assume that $n \geq 2$ and $E \subset \mathbb{R}^n$ is a set. A *ridge function* on $E$ is a function of the form $\varphi(\mathbf{a} \cdot \mathbf{x})$, where $\mathbf{x} = (x_1, \dots, x_n) \in E$, $\mathbf{a} = (a_1, \dots, a_n) \in \mathbb{R}^n \setminus \{\mathbf{0}\}$, $\mathbf{a} \cdot \mathbf{x} = \sum_{j=1}^{n} a_j x_j$ and $\varphi$ is a real-valued function defined on $\Delta(\mathbf{a}) = \{\mathbf{a} \cdot \mathbf{x} : \mathbf{x} \in E\}$. On a set $E$, consider a sum of ridge functions

$$f(\mathbf{x}) = \sum_{i=1}^{m} \varphi_i(\mathbf{a}^i \cdot \mathbf{x}). \quad (1)$$

We assume that the vectors $\mathbf{a}^i$ are pairwise noncollinear. Set $\Delta_i = \Delta(\mathbf{a}^i)$. A closed convex set $E \subset \mathbb{R}^n$ such that $int(E) \neq \emptyset$ is called a *convex body*. Suppose that $f \in C(E)$. We study some properties of the functions $\varphi_i$ under this assumption and prove that if all of the $\varphi_i$ belong to some wide class $B$ (for instance, the union of the set of locally bounded functions and the set of Lebesgue measurable functions) then each of the $\varphi_i$ is continuous on the interior of $\Delta_i$. Then we obtain the condition in terms of the modulus of continuity of $f$, which provides the existence of finite limits of the functions $\varphi_i$ at the boundary points of $\Delta_i$.

# On the Norms of the Integral Means of the Bochner-Riesz Sums

O. Kuznetsova

(Institute of Applied Mathematics and Mechanics, Donetsk)

kuznetsova.olga.15@mail.ru

The report deals with the Bochner-Riesz sums

$$S_R^{\alpha}(f,x) = \sum_{\|k\| \leqslant R} \left(1 - \|k\|^2 \setminus R^2\right)^{\alpha} \hat{f}(k) e^{ik\cdot x}$$

of a periodic function in $m$ variables and the strong integral means

$$\left( \left( \int_0^R |S_r^{\alpha}(f,x)|^p \, dr \right) / R \right)^{1/p}$$

of these sums for $p \geqslant 1$ and $0 < \alpha \leqslant \frac{m-1}{2}$.

We have established [1] estimates for growth as $R \to \infty$ of the norms of corresponding operators, i. e., of the quantities

$$\sup_{|f| \leqslant 1} \left( \left( \int_0^R |S_r^{\alpha}(f,0)|^p \, dr \right) / R \right)^{1/p} \qquad (*)$$

for $0 < \alpha < \frac{m-1}{2}$.

The exact growth order as $R \to \infty$ of $(*)$ for integral means of spherical Fourier sums ($\alpha = 0$) was established in [2].

The boundedness of the norms $(*)$ for the critical order $\alpha = \frac{m-1}{2}$ is proved in [3] ([4] for $p = 2$).

## References

[1] O.I. Kuznetsova, On the norms of the integral means of Bochner-Riesz sums, Trudy IPMM, 30, 91–95 (2016).

[2] O.I. Kuznetsova, A.N. Podkorytov, On the norms of the integral means of spherical Fourier sums, Math. Notes 96(5), 55–62 (2014).

[3] Wang Kunyang, Gavin Broun, Aproximation by Bochner-Riesz means and Hardy summability, J. Beijing Normal Univ. (Natural Science), 30(2), 163–169 (1994).

[4] Wang Kunyang, Strong uniform approximation by Bochner–Riesz means, Multivariate approximation IV, Proceedings of conference of multivariate approximation theory, Oberwolfach, West Germany, 337–342 (1989).

## Unconditional Convergence for Wavelet Frame Expansions

E. Lebedeva

(Saint Petersburg State University, Russia)

ealebedeva2004@gmail.com

We study unconditional convergence for wavelet frame expansions in $L_p(\mathbb{R})$.

Let $\{\psi_{j,k}\}_{(j,k)\in\mathbb{Z}^2}$, $\{\tilde{\psi}_{j,k}\}_{(j,k)\in\mathbb{Z}^2}$ be dual wavelet frames in $L_2(\mathbb{R})$, let $\eta$ be an even, bounded, decreasing on $[0,\infty)$ function such that

$$\int_0^\infty \eta(x)\ln(1+x)\,dx < \infty,$$

and $|\psi(x)|, |\tilde{\psi}(x)| \le \eta(x)$. Then the series $\sum_{j,k\in\mathbb{Z}}(f, \tilde{\psi}_{j,k})\psi_{j,k}$ is unconditional convergent in $L_p(\mathbb{R})$, $1<p<\infty$.

## Toeplitz and Hankel Operators in Non-Algebraic Setting

K. Lesnik
(Poznan University of Technology, Poland)
klesnik@vp.pl

The theory of Toeplitz and Hankel operators is widely investigated and well developed, but mainly in the algebraic setting, i.e. when they act from one to the same space. In contrast to this, we are interested in a non-algebraic situation, i.e. when these operators act between distinct Hardy spaces. We will present non-algebraic analogues of Brown–Halmos and Nehari theorems. Moreover, we will discuss some basic properties of such operators and explain how they are related with the problem of factorization of functions and with properties of spaces of pointwise multipliers.

## Comparing the Degrees of Unconstrained and Constrained Approximation by Polynomials

D. Leviatan
(Tel Aviv University, Israel)
leviatan@tauex.tau.ac.il

It is quite obvious that one should expect that the degree of constrained approximation be worse than the degree of unconstrained approximation. However, it turns out that in certain cases we can deduce the behavior of the degrees of the former from information about the latter.

Let $E_n(f)$ denote the degree of approximation of $f \in C[-1,1]$, by algebraic polynomials of degree $< n$, and assume that we know that for some $\alpha > 0$ and $N \geq 1$,

$$n^{\alpha} E_n(f) \leq 1, \quad n \geq N.$$

Suppose that $f \in C[-1,1]$, changes its monotonicity or convexity $s \geq 0$ times in $[-1,1]$ ($s = 0$ means that $f$ is monotone or convex, respectively). We are interested in what may be said about its degree of approximation by polynomials of degree $< n$ that are comonotone or coconvex with $f$. Specifically, if $f$ changes its monotonicity or convexity at $Y_s := \{y_1, \dots, y_s\}$ ($Y_0 = \emptyset$) and the degrees of comonotone and coconvex approximation are denoted by $E_n^{(q)}(f, Y_s)$, $q = 1, 2$, respectively. We investigate when can one say that

$$n^{\alpha} E_n^{(q)}(f, Y_s) \leq c(\alpha, s, N), \quad n \geq N^*,$$

for some $N^*$. Clearly, $N^*$, if it exists at all (we prove it always does), depends on $\alpha$, $s$ and $N$. However, it turns out that for certain values of $\alpha$, $s$ and $N$, $N^*$ depends also on $Y_s$ and, in some cases, even on $f$ itself and this dependence is essential.

## The Fourier Transform of a Function of Bounded Variation: Symmetry and Asymmetry

E. Liflyand
(Bar-Ilan University, Ramat-Gan, Israel)
liflyand@gmail.com

New relations between the Fourier transform of a function of bounded variation and the Hilbert transform of its derivative are revealed. The main result is an asymptotic formula for the **cosine** Fourier transform.

Such relations have previously been known only for the sine Fourier transform. To prove the mentioned result, not only a different space is considered but also a new way of proving such theorems is applied.

Interrelations of various function spaces are studied in this context. The obtained results are used for proving new estimates for the Fourier transform of a radial function and completely new results on the integrability of trigonometric series.

## Pseudo-Amenability of Some Weighted Semigroup Algebras

A. Mahmoodi
(Islamic Azad University, Tehran, Iran)
a_mahmoodi@iauctb.ac.ir

Let $S$ be a semigroup and let $\omega : S \longrightarrow [1, +\infty)$ be a function such that $\omega(st) \leq \omega(s)\omega(t)$ for $s, t \in S$. Then $\ell^1(S, \omega)$ is a Banach algebra of all functions $f : S \longrightarrow \mathbb{C}$ with the norm $||f||_\omega = \sum_{s \in S} |f(s)| \; \omega(s) < +\infty$, and with the product defined by $\delta_s \star \delta_t = \delta_{st}$. A semigroup $S$ is called a left zero semigroup if $st = s$ for each $s, t \in S$. In this note, we study pseudo-amenability of $\ell^1(S, \omega)$, where $S$ is a left zero semigroup.

## On a Lower Bound of Polynomial Norms in a Hilbert Space

D. Martirosyan
(American University of Armenia, Armenia)
davit_martirosyan@edu.aua.am

Let $H$ be a Hilbert space over the field of complex numbers. For a linearly independent system $\{e_k\}_{k=1}^n \subset H$ consider $E_n$, the family of polynomials $\sum_{k=1}^n \alpha_k e_k$, where the coefficients are chosen so that $|\alpha_k| \leq 1$ for $k = 1, 2, ..., n$.

The presented theorem establishes a lower estimate for norms of general linear combinations of $\{e_k\}_{k=1}^n$ by the $(n-1)$th Kolmogorov width of $E_n$ in $H$. The latter is denoted by $d_{n-1}(E_n, H)$ (see [1], page 399).

**Theorem 1.** *For any collection of coefficients $\{a_k\}_{k=1}^n$, the inequality*

$$|| \sum_{k=1}^{n} a_k e_k ||_H \geq d_{n-1}(E_n, H) \cdot \max_{1 \leq k \leq n} |a_k|$$

*holds.*

The proof is based on estimations of norms of biorthogonal systems and uses one result from [2].

# References

[1] G. G. Lorentz, M. V. Golitschek, Y. Makovoz, Constructive Approximation: Advanced Problems, Springer Verlag, Berlin, 1996.

[2] M. Martirosyan, S. Samarchyan, On geometrically decreasing Kolmogorov $n$-widths in Hilbert spaces, Abstracts of International Conference Harmonic Analysis and Approximations, IV, 2008, pp. 90-91.

## The Kahane–Salem–Zygmund Inequalities for Random Polynomials

M. Mastylo
(Adam Mickiewicz University, Poland)
mastylo@amu.edu.pl

We will discuss variants of the Kahane–Salem–Zygmund inequalities for the expectation of the supremum norm of homogeneous Bernoulli polynomials on the unit ball of a Banach space. We combine ideas from stochastic processes and interpolation methods to control increments of a Rademacher process in an Orlicz space via entropy integrals. The talk is based on a joint work with R. Szwedek.

## Riesz Basis Criteria for Families of Dilated Periodic Functions

H. Melkonian
(Heriot-Watt University)

Consider a periodic function $f$, such that its restriction to the unit segment lies in the Banach space $L^2 = L^2(0,1)$. Denote by $S$ the family of dilations $f(nx)$ for all $n$ positive integer. The purpose of this talk is to discuss the following question: When does $S$ form a Riesz basis of $L^2$?

In this talk, we will present a new *mutli-term* criteria for determining Riesz basis properties of $S$ in $L^2$. This method was established in [L. Boulton, H. Melkonian; arXiv: 1708.08545 J. (2017)] and it relies on a general framework developed by Hedenmalm, Lindqvist and Seip about 20 years ago, which turns the basis question into one about the localisation of the zeros and poles of a corresponding analytic multiplier. Our results improve upon various criteria formulated previously, which give sufficient conditions for invertibility of the multiplier in terms of sharp estimates on the Fourier coefficients.

We will then examine the application of these criteria in the case of $f$ being the $p$-trigonometric functions. These functions arise naturally in the context of the non-linear eigenvalue problem associated to the one-dimensional $p$-Laplacian in the unit segment. These results improve upon those of [D. E. Edmunds, P. Gurka, J. Lang, *J. Math. Anal. Appl.* 420 (2014)] and [L. Boulton, H. Melkonian J. Math. Anal. Appl., 444 (2016)].

## The Gibbs Phenomenon for General Franklin Systems

V. Mikayelyan
(Yerevan State University, Armenia)
mik.vazgen@gmail.com

The Gibbs Phenomenon discovered by Henry Wilbraham in 1848 and rediscovered by Josiah Willard Gibbs in 1899, is the peculiar manner in which the Fourier series of some function behaves at a jump discontinuity. The n-th partial sum of the Fourier series has large oscillations near the jump, which might increase the maximum of the partial sum above that of the function itself. The overshoot does not die out as n increases, but approaches a finite limit.

The general Franklin system corresponding to a given dense sequence of points $T = (t_n, n \geq 0)$ in $[0, 1]$ is a sequence of orthonormal piecewise linear functions with knots from $T$, that is, the nth function from the system has knots $t_0, \ldots, t_n$.

The Gibbs Phenomenon has been studied for Fourier series with respect to several famous orthonormal systems (see [1]-[4]). We studied the Gibbs phenomenon for general Franklin systems. We proved that the Gibbs phenomenon occurs for almost all points of $[0,1]$.

# References


[1] N. K. Bari, *Trigonometric series* (in Russian), Moscow: Gos. Izdat. fiz.-mat. Literatury, 1961.

[2] A. M. Zubakin, The Gibbs phenomenon for multiplicative systems of Walsh or Vilenkin-Dzhofarli type, *Siberian Math. J.*, **12** (1971), 147–157

[3] L.A. Balashov, V.A. Skvortsov, Gibbs constants for partial sums of Fourier-Walsh series and their $(C,1)$ means, *TrudyMat. Inst. Steklov.*, **164** (1983), 37–48.

[4] O.G. Sargsyan, On the convergence and Gibbs phenomenon of Franklin series, *Journal of Contemporary Mathematical Analysis*, **31 (1)** (1996), 61–84.


## Selected Aspects of Complex Symmetric Operators


P. Mleczko

(Adam Mickiewicz University in Pozna, Poland)

pml@amu.edu.pl


In the course of a talk I will discuss interpolation properties of complex symmetric operators on Hilbert spaces and show applications to the study of Toeplitz operators on weighted Hardy–Hilbert spaces of analytic functions on the unit disc. The talk is based on a joint work with Radosław Szwedek from Adam Mickiewicz University in Poznan.

# Uniqueness Theorems for Vilenkin and Generalized Haar Systems

K. Navasardyan
(Yerevan State University, American University of Armenia, Armenia)
knavasard@ysu.am

Let $\{p_k\}$ be a sequence of natural numbers and $p_k \geq 2$. In the sequel $\{f_n(x)\}_{n=0}^{\infty}$ will be either the generalized Haar system or Vilenkin system, generated by the sequence $\{p_k\}$. For this system a new regular summation method is defined and using that method uniqueness Theorems are proved.

Let $\sum\limits_{n=0}^{\infty} a_n f_n(x)$ be a series and let $S_m(x) := \sum\limits_{n=0}^{m} a_n f_n(x)$ be its partial sum. Denote $m_0 = 1$, $m_k = p_k m_{k-1}$.

The following Theorem is proved.

**Theorem 1.** *If the partial sums $S_{m_k}(x)$ converge in measure to some $f(x)$, as $k \to \infty$, and for some increasing sequence $\lambda_\nu \to \infty$ the condition*

$$\lim_{\nu\to\infty} \lambda_\nu \cdot \text{mes}\left\{x \in [0,1) : \sup_m |S_m(x)| > \lambda_\nu \right\} = 0$$

*holds, then $a_n = \lim\limits_{\nu\to\infty} \int_0^1 [f(x)]_{\lambda_\nu} \overline{f_n(x)} dx$ for every natural number $n$, where*

$$[g(x)]_\lambda := \begin{cases} g(x), & \text{if } |g(x)| \leq \lambda, \\ 0, & \text{if } |g(x)| > \lambda. \end{cases}$$

Note that a similar Theorem, when $\{p_k\}$ is bounded, was proved by Kostin V.

Uniqueness Theorems for multiple series are also considered.

# On the Axioms of Hypergroup Over a Group

Sh. Navasardyan
(Yerevan State University, Armenia)
navasardyanshant@gmail.com

The concept of hypergroup over group arises when one tries to extend the concept of quotient group in case of any subgroup of the given group. This concept unifies and generalizes the concepts of the group, of the field and of the vector space over field. A hypergroup over group is a pair $(M, H)$, where M is a set and H is a group, together with structural mappings

- ($\Phi$) $\Phi : M \times H \to M, \qquad \Phi(a, \alpha) := a^{\alpha},$
- ($\Psi$) $\Psi : M \times H \to H, \qquad \Psi(a, \alpha) := {}^{a}\alpha,$
- ($\Xi$) $\Xi : M \times M \to M, \qquad \Xi(a, b) := [a, b],$
- ($\Lambda$) $\Lambda : M \times M \to H, \qquad \Lambda(a, b) := (a, b)$

which satisfy the following conditions:

$P1)$ $\Xi$ is a binary operation such that

$(i)$ $(M, \Xi)$ is a right quasigroup, i. e. any equation $[x, a] = b$ with elements $a, b \in M$ has a unique solution in $M$;

$(ii)$ $\Xi$ has a left neutral element $o$, i.e. $[o, a] = a$ for any $a \in M$.

$P2)$ The mapping $\Phi$ is an action of $H$ on $M$, that is

$(i)$ $(a^{\alpha})^{\beta} = a^{\alpha \cdot \beta}$ for any $\alpha, \beta \in H$, $a \in M$;

$(ii)$ $a^{\varepsilon} = a$ for any element $a \in M$, where $\varepsilon$ is the neutral element of the group $H$.

$P3)$ For any element $\alpha \in H$, there exists an element $\beta \in H$ such that $\alpha = {}^{o}\beta$.

$P4)$ The following identities hold:

- $(A1) \quad {}^{a}(\alpha \cdot \beta) = {}^{a}\alpha \cdot {}^{a^{\alpha}}\beta,$
- $(A2) \quad [a,b]^{\alpha} = [a^{{}^{b}\alpha}, b^{\alpha}],$
- $(A3) \quad (a,b) \cdot {}^{[a,b]}\alpha = {}^{a}({}^{b}\alpha) \cdot (a^{{}^{b}\alpha}, b^{\alpha}),$
- $(A4) \quad [[a,b],c] = [a^{(b,c)}, [b,c]],$
- $(A5) \quad (a,b) \cdot ([a,b],c) = {}^{a}(b,c) \cdot (a^{(b,c)}, [b,c]).$

Our work consists of three parts. In the first part, by constructing proper models, we show that the axioms $(P1)$, $(P2)$, $(P3)$, $(A1)$, $(A2)$, $(A3)$, $(A4)$, $(A5)$ of hypergroup over group are independent.

In the second part we prove a theorem which claims that in some special cases, namely when the action $\Phi$ is faithful (i.e. there is no non-trivial element $\alpha \in H$ such that $a^{\alpha} = a$ for any element $a \in M$) and $o^{\alpha} = o$ for all elements $\alpha \in H$, the axioms $(P3), (A1), (A3), (A5)$ follow from $(P1), (P2), (A2), (A4)$.

In the third part we define the concept of faithful extension of a group by right quasigroup with left neutral element as follows. Let $H$ be a group and $(M, \Xi)$ be a right quasigroup with a left neutral element $o$. Let $G$ be a group which has a subgroup $\tilde{H}$ isomorphic to $H$ and let $\tilde{M}$ be a (right) transversal of $\tilde{H}$ in $G$ (i.e. $|\tilde{M} \cap \tilde{H}g| = 1$ for all elements $g \in G$). Then every element $g \in G$ has a unique decomposition $g = \tilde{\alpha} \cdot \tilde{a}$, where $\tilde{\alpha} \in \tilde{H}, \tilde{a} \in \tilde{M}$, hence for any elements $\tilde{\alpha} \in \tilde{H}, \tilde{a}, \tilde{b} \in \tilde{M}$ there is a unique decomposition (so the mappings $\tilde{\Phi}, \tilde{\Psi}, \tilde{\Xi}, \tilde{\Lambda}$ will be well-defined):

- $\tilde{a} \cdot \tilde{\alpha} = \tilde{\Psi}(\tilde{a}, \tilde{\alpha}) \cdot \tilde{\Phi}(\tilde{a}, \tilde{\alpha})$,
- $\tilde{a} \cdot \tilde{b} = \tilde{\Lambda}(\tilde{a}, \tilde{b}) \cdot \tilde{\Xi}(\tilde{a}, \tilde{b})$.

*Definition:* We say that the group $G$ is an extension of $H$ by $(M, \Xi)$, if $G$ has a subgroup $\tilde{H}$ isomorphic to $H$ and there exists a transversal $\tilde{M}$ such that the gruppoid $(\tilde{M}, \tilde{\Xi})$ is isomorphic to $(M, \Xi)$. We call this extension faithful if $\tilde{\Phi}$ is a faithful action of $\tilde{\Phi}$ on $\tilde{M}$ and which fixes the left neutral element of $(\tilde{M}, \tilde{\Xi})$ for all elements $\tilde{\alpha} \in \tilde{H}$.

And at the end of the third part for a given group $H$ and right quasigroup $(M, \Xi)$ with left neutral element, we describe all faithful extensions of $H$ by $(M, \Xi)$ in terms of hypergroups over group. Moreover, in the finite case we provide an efficient algorithm for finding them.

# References


[1] Dalalyan S. H., *On hypergroups, prenormal subgroups and simplest groups*. Conf. dedicated to 90-anniversary of M. M. Jrbashyan, Yerevan, 2008, p. 12-14. (Russian)

[2] Dalalyan S. H., *Hypergroups over the group and extensions of a group*, Second Int. Conf. Mathematics in Armenia, 24-31 Aug 2013, Tsaghkadzor (Armenia) , Abstracts, (2013), p 111. (Russian)

[3] Dalalyan S. H., *Hypergroups over the group and generalizations of Schreier's theorem on group extensions*, arXiv:1403.6134 [math.GR].

## On Some Relations Between Ergodicity and Controllability for Stochastic Systems

V. Nersesyan
(Université de Versailles, France)
vahagn.nersesyan@uvsq.fr

In this talk, we will discuss some recent results on the long-time behaviour of solutions of PDEs perturbed by a random force. We will show that under some generic controllability hypotheses (satisfied for Navier-Stokes and Ginzburg-Landau equations), there is a unique invariant measure which attracts exponentially all the solutions. This is a joint work with S. Kuksin and A. Shirikyan.

## Reconstruction of Convex Bodies by Covariogram

V. Ohanyan
(Yerevan State University, Armenia)
victoohanyan@ysu.am

Let $\mathbf{R}^n$ $(n \geq 2)$ be the $n$-dimensional Euclidean space, $\mathbf{D} \subset \mathbf{R}^n$ be a bounded convex body with inner points, and $V_n$ be the n-dimensional Lebesgue measure in $R^n$. The function $C(\mathbf{D}, h) = V_n(\mathbf{D} \cap (\mathbf{D} + h)), \quad h \in \mathbf{R}^n$, is called the covariogram of the body $\mathbf{D}$. Here $\mathbf{D} + h = \{x + h,\, x \in \mathbf{D}\}$. Very little is known regarding the covariogram problem when the space dimension is greater than 2. It is known that centrally symmetric convex bodies in any dimension, are determined by their covariogram up to translations. For n=3 the problem is open. Let $\Pi r_{\mathbf{u}^\perp} \mathbf{D}$ be the orthogonal projection of $\mathbf{D}$ onto the hyperplane $\mathbf{u}^\perp$ (here $\mathbf{u}^\perp$ stands for the hyperplane with normal $\mathbf{u}$, passing through the origin). A random line which is parallel to $\mathbf{u}$ and intersects $\mathbf{D}$ has an intersection point (denoted by $x$) with $\Pi r_{\mathbf{u}^\perp}\mathbf{D}$. We can identify the points of $\Pi r_{\mathbf{u}^\perp}\mathbf{D}$ and the lines which intersect $\mathbf{D}$ and are parallel to $\mathbf{u}$. Assuming that the intersection point $x$ is uniformly distributed over the convex body $\Pi r_{\mathbf{u}^\perp}\mathbf{D}$, we can define the following distribution function. The function

$$F(\mathbf{u}, t) = \frac{V_{n-1}\{x \in \Pi r_{\mathbf{u}^\perp}\mathbf{D} : V_1(g(\mathbf{u}, x) \cap \mathbf{D}) < t)\}}{b_{\mathbf{D}}(\mathbf{u})}$$

is called orientation-dependent chord length distribution function of $\mathbf{D}$ in direction $\mathbf{u}$ at point $t \in R^1$, where $g(\mathbf{u}, x)$ is the line which is parallel to $\mathbf{u}$ and intersects $\Pi r_{\mathbf{u}^\perp}\mathbf{D}$ at point $x$ and $b_{\mathbf{D}}(\mathbf{u}) = V_{n-1}(\Pi r_{\mathbf{u}^\perp}\mathbf{D})$. Denote by $\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D})$ the probability that the random segment $L(\mathbf{u}, \omega)$ (of fixed length $l$ and direction $\mathbf{u}$) entirely lies in the body $\mathbf{D}$.

**Proposition 1.** (see [1]). The probability $\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D})$ in terms of the distribution function $F(\mathbf{u}, t)$ has the following form:

$$\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D}) = \frac{V_n(\mathbf{D}) - l\, b_{\mathbf{D}}(\mathbf{u}) + b_{\mathbf{D}}(\mathbf{u})) \int_0^l F(\mathbf{u}, z)\, dz}{V_n(\mathbf{D}) + l\, b_{\mathbf{D}}(\mathbf{u})},$$

while in terms of the covariogram of the body $\mathbf{D}$ has the form:

$$\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D}) = \frac{C(\mathbf{D}, \mathbf{u}, l)}{V_n(\mathbf{D}) + l\, b_{\mathbf{D}}(\mathbf{u})}.$$

# References

[1] N. G. Aharonyan, V. K. Ohanyan, Calculation of geometric probabilities using Covariogram of convex bodies. *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 53 (2), pp. 112–120, 2018.

## Around Cantor Uniqueness Theorem

A. Olevskii
(Tel Aviv University, Israel)
olevskii@yahoo.com

I'll discuss the following problem formulated in early 1960-s by A.A. Talalyan and P.L. Ul'yanov: can a (non-trivial ) trigonometric series

$$\sum c(n)e^{int}, \qquad c(n) = o(1),$$

have a subsequence of partial sums converging to zero everywhere ?

This is a joint work with G. Kozma.

## On the Differentiation of Integrals With Respect to Translation Invariant Convex Density Bases

G. Oniani
(Akaki Tsereteli State University, Georgia)
oniani@atsu.edu.ge

For a translation invariant convex density basis $B$ it is shown that its Busemann-Feller extension $B_{\mathrm{BF}}$ has close to $B$ properties, namely, $B_{\mathrm{BF}}$ differentiates the same class of non-negative functions as $B$, moreover, the integral of an arbitrary non-negative function $f \in L(\mathbb{R}^n)$ at almost every point $x \in \mathbb{R}^n$ has one and the same type limits of indeterminacy with respect to the bases $B$ and $B_{\mathrm{BF}}$. Using this theorem the restriction of Busemann-Fellerity is removed from some known results. One of such type applications allows us to give an answer to a question of Guzmán on the limits of indeterminacy of indefinite integrals for the class of homothecy invariant convex bases.

## Spline Characterizations the Radon-Nikodym-Property

M. Passenbrunner
(Johannes Kepler Univeristy Linz, Austria)
markus.passenbrunner@jku.at

A Banach space $X$ is said to have the Radon-Nikodým-property (RNP) if, for measures with values in $X$, the Radon-Nikodým theorem is true, i.e. if for every positive measure $\mu$ and for every $\mu$-continuous measure $\nu$ of bounded variation with values in $X$, there exists an integrable function $f$ with values in $X$ so that

$$\nu(A) = \int_A f d\mu$$

for every measurable set $A$. The RNP can be characterized in terms of martingale convergence, i.e., for a Banach space $X$, all $L^1$-bounded martingales $(f_n)$ with values in $X$ converge almost surely if and only if $X$ has the RNP.

In this talk, we give a similar characterization of the RNP in terms of general polynomial spline sequences instead of martingales. Part of this work is joint with P.F.X. Müller.

# Weighted spaces of functions harmonic in the unit ball

A. Petrosyan
(Yerevan State University, Armenia)
apetrosyan@ysu.am

In the work [1] we introduce the Banach spaces $h_\infty(\varphi)$, $h_0(\varphi)$, and $h^1(\psi)$ of functions harmonic in the unit ball $B \subset \mathbb{R}^n$. These spaces depend on weight functions $\varphi$, $\psi$.

The positive and continuous on $[0, 1)$ function $\varphi$ is called normal, if there are three constants $0 < a < b$ and $0 \le r_0 < 1$ such that

$$\frac{\varphi(r)}{(1-r)^a} \quad \text{decreases in} \quad r_0 \le r < 1 \quad \text{and} \quad \lim_{r\to 1^-} \frac{\varphi(r)}{(1-r)^a} = 0,$$
$$\frac{\varphi(r)}{(1-r)^b} \quad \text{increases in} \quad r_0 \le r < 1 \quad \text{and} \quad \lim_{r\to 1^-} \frac{\varphi(r)}{(1-r)^b} = \infty.$$

The functions $\{\varphi, \psi\}$ will be called a normal pair if $\varphi$ is normal and if there is a number $\alpha$ (the pair index) such that

$$\varphi(r)\psi(r) = (1-r^2)^\alpha, \quad 0 \le r < 1, \quad \text{and} \quad \int_0^1 \psi(r)\, dr < \infty.$$

We prove that if $\varphi$ and $\psi$ form a normal pair, then $h^1(\psi)^* \sim h_\infty(\varphi)$ and $h_0(\varphi)^* \sim h^1(\psi)$, i.e. $h^1(\psi)$ represents the intermediate space, the dual of $h_0(\varphi)$ and the predual of $h_\infty(\varphi)$.

## References

[1] A. I. Petrosyan, K. L. Avetisyan, Weighted spaces of functions harmonic in the unit ball, Proceedings of the Yerevan State University, Physical and Mathematical Sciences, 2017, Volume 51, Issue 1, p. 3-7.

## A Boundary Property of Some Subclasses of Functions of Bounded Type in the Half-Plane

J. Restrepo
(Rostov on Don, Russia)
cocojoel89@yahoo.es

We give a construction of the half-plane analog of the part of the factorization theory of M.M. Djrbashian - V.S. Zakaryan [1,2], where Djrbashian's generalized fractional integral was used to establish the descriptive representations and boundary properties of meromorphic in the unit disc functions of the classes $N\{\omega\}$ contained in the Nevanlinna class $N$ of functions of bounded type.

Some results of nearly the same type are obtained for several weighted classes of meromorphic in the upper half-plane functions with bounded Tsuji characteristics by the application of the Laplace transform along with an Hadamard-Liouville type generalized integro-differential operator with an unbounded integration contour, which becomes the Liouville integro-differentiation in a particular case.

# References

[1] *M.M. Djrbashian and V.S. Zakaryan* Classes and Boundary Properties of Functions Meromorphic in the Disc. Nauka, Moscow (1993).

[2] *M.M. Djrbashian* Theory of factorization and boundary properties of functions meromorphic in the disc. Proc. International Congress of Mathematicians (Vancouver 1974), Vol. 2, Canad. Math. Congress, Montreal (1975), 197–202.

# Area Integral and Polynomial Approximations of Holomorhic Functions In $\mathbb{C}^n$

A. Rotkevich
(Saint Petersburg State University, Russia)
rotkevichas@gmail.com

Let $G \subset \mathbb{C}$ be a Radon domain, for $z \in \partial G$ we consider a sector $S(z) = \left\{\xi \in G : \mathrm{dist}(\xi, \partial G) \geq \frac{1}{2}|\xi - z|\right\}$. It is well known, that for function $f$ holomorphic on $G$ one has $\left\|I_f\right\|_{L^p(\partial G)} \leq c(p,G)\left\|f\right\|_{L^p(\partial G)}$, $1 < p < \infty$, with some constant $c(p,G)$, where $I_f$ is an area-integral $I_f(z)^2 = \int\limits_{S(z)} |f'(\xi)|^2 \, d\mu(\xi)$ and $d\mu$ is the Lebesgue measure on $\mathbb{C}$.

In 1984 E.M. Dynkin applied this inequality to describe Smirnov spaces $E^l_p(G)$ by pseudoanalytic continuation and this construction essentially led to a characterization of these spaces by polynomial approximations on $\partial G$. There are many many generalizations of the above area-integral inequality for holomorphic functions on regular domains in $\mathbb{C}^n$ by P. Ahern, J. Bruna (1988), A. Nagel, E.M. Stein, S. Wainger (1981), G. Sardine (1993), S. Krantz and S.Y. Lee (1997). We consider a generalization of these estimates to functions generated by conjugate Cauchy-Leray-Fantappiè integral and defined on the complement of a bounded domain in $\mathbb{C}^n$ and discuss a role of these inequalities in characterization of spaces of holomorphic functions.

# On Approximation Processes Defined by the Cosine Operator Function in a Banach Space

A. Saksa
(Tallinn University of Technology, Estonia)
anna.saksa@ttu.ee

In our presentation we introduce the cosine-type approximation processes in abstract Banach space $X$. Suppose with each $f \in X$ one may associate its unique Fourier series expansion with the Fourier partial sums operator

$$S_n f = \sum_{k=0}^{n} P_k f.$$

Then we define approximation operators as follows.

**Definition.** The cosine-type operators $U_{n,h,\mathbf{a}} : X \to X$ are defined by

$$U_{n,h,\mathbf{a}} g := \sum_{k=0}^{m} a_k C_{kh} \left( S_n g \right), \; h \geq 0, \; g \in X$$

where $\mathbf{a} = (a_0, ..., a_m) \in \mathbb{R}^{m+1}, \; m \geq 1$ and

$$\sum_{k=0}^{m} a_k = 1.$$

Here the cosine operator function $C_h \in [X]$ $(h \geq 0)$ is defined by the properties:

1. $C_0 = I$(identity operator),
2. $C_{h_1} \cdot C_{h_2} = \frac{1}{2}(C_{h_1+h_2} + C_{|h_1-h_2|})$,
3. $\|C_h f\| \leq T\|f\|$, the constant $T > 0$ is not depending on $h > 0$.

The historical roots of these processes go back to W. W. Rogosinski ([2]) in 1926. Our given new definitions use a cosine operator functions concept ([1]) in two ways. First, the approximation operators are defined via the cosine operator function, and second, the modulus of continuity is defined via the same cosine operator function. We proved that in presented setting the cosine-type operators possess the order of approximation, which coincide with results known in trigonometric approximation.

Also applications for different type of approximations will be given.

The work is co-authored with A. Kivinukk.

## References


[1] Kivinukk, A., Saksa, A., On approximation by Blackman- and Rogosinski-type Operators in Banach Space. *Proc. of the Estonian Acad. of Sci.*, 2016, **65**, 205-219.

[2] Rogosinski, W. W. *Reihensummierung durch Abschnittskoppelungen* Math. Z., 1926, **25**, 132-149.


## The Hardy's Theorem and Rotation


P. Sarathi Patra

(IIT Hyderabad, India)

ma13p1003@iith.ac.in


In this talk we will present a version of Hardy's theorem. Lakey and Hogan generalized the Hardy's theorem (a quantitative uncertainty principle) by considering the decay of $f$ and $\hat{f}$ along arbitrary rays in the plane $\mathbb{C}$. We give an alternative proof of Lakey and Hogan's theorem by using Hermite semigroup. The proof using Hermite semigroup is relatively simpler.

This idea of using Hermite semigroup helped us to do a further generalization of Lakey and Hogan's theorem by replacing the Fourier transform with Dunkl transform associated to the reflection group $G = \mathbb{Z}_2^d$ in $\mathbb{R}^d$. We give a version of Hardy's theorem for rotation by assuming the decay in the Hermite coefficient.

## On the Structure of Universal Functions for Weighted Spaces $L^p_\mu[0,1],\ p > 1$


A. Sargsyan

(Russian-Armenian University, Armenia)

asargsyan@ysu.am


The talk is devoted to the structure of functions (from Lusin's correction theorem point of view), which are universal for weighted spaces $L^p_\mu[0,1],\ p > 1$ with respect to Walsh system. The following result will be discussed:

**Theorem.** *For each number $\varepsilon \in (0,1)$ there exist a measurable set $E_\varepsilon \subset [0,1]$ with measure $|E_\varepsilon| > 1 - \varepsilon$ and a weight function $0 < \mu(x) \leq 1$ with condition $\mu(x) = 1$ on $E_\varepsilon$, so that for every function $f \in L^1[0,1]$ one can find a function $\tilde{f} \in L^1[0,1]$, which coincides with $f$ on $E_\varepsilon$ and is universal for each space $L^p_\mu[0,1]$, $p > 1$ with respect to the Walsh system in the sense of signs–subseries of Fourier series.*

# References


[1] Sargsyan A. A., "Quasi-universal Fourier–Walsh series in $L_p[0,1], p > 1$ classes", Math. Notes, 104:2(2018).

[2] Grigoryan M. G., Sargsyan A. A., The structure of universal functions for $L^p$ spaces, $p \in (0,1)$, Sbornik: Mathematics, 209:1(2018), 37–57.

[3] Grigoryan M. G., Grigoryan T. M., Sargsyan A. A., On the universal function for weighted spaces $L^p_\mu[0,1]$, $p \geq 1$, Banach J. Math. Anal., 12:1(2018), 104–125.

[4] Sargsyan A. A., Grigoryan M. G., Universal function for a weighted space $L^1_\mu[0,1]$, Positivity, 21(2017), 1457–1482.

[5] Grigoryan M. G., Sargsyan A. A., On the universal function for the class $L^p[0,1], p \in (0,1)$, Journal of Func. Anal., 270(2016), 3111–3133.

[6] Grigoryan M. G., Sargsyan A. A., On existence of a universal function for $L^p[0,1], p \in (0,1)$, Siberian Math. Journal, 57:5(2016), 796–808.

# On the Convergence and Behavior of Coefficients of Fourier-Vilenkin Series

S. Sargsyan

(Yerevan State University, Armenia)

stepansargsyan@ysu.am

In this talk we will present the following result regarding to both bounded and some type of unbounded Vilenkin systems:

**Theorem 1.** *For any $\varepsilon \in (0,1)$, there exists a measurable set $E \subset [0,1)$ of measure larger than $1-\varepsilon$ such that for any function $f \in L^1[0,1)$, it is possible to find a function $g \in L^1[0,1)$ coinciding with $f$ on $E$, Fourier series of $g$ with respect to the Vilenkin system of exponential growth type is convergent in $L^1$-norm and the absolute values of non zero Fourier coefficients of $g$ are monotonically decreasing.*

For each Vilenkin system, particularly of unbounded type, we show (see [2]):

**Theorem 2.** *Let $\{W_k(x)\}_{k=0}^{\infty}$— be either unbounded or bounded Vilenkin system. Then for each $0 < \epsilon < 1$ there exists a measurable set $E \subset [0,1)$ of* *measure $|E| > 1 - \varepsilon$ such that for any function $f \in L^1[0,1)$ there exists a function $g \in L^1[0,1)$ such that $f(x) = g(x)$ if $x \in E$ and the elements of sequence $\{|c_k(g)|, k \in spec(g)\}$ are monotonically decreasing.*

Note that the following problems remain open:

**Question 1.** *Is the Fourier series of a function from $L^2[0,1)$ with respect to the unbounded Vilenkin systems convergent almost everywhere or not?*

**Question 2.** *Is it possible to choose a modified function in Theorem 1 and Theorem 2 such that also Fourier-Vilenkin series of that function are almost everywhere convergent?*

# References


[1] Grigoryan M G, Sargsyan S A, On the L1-convergence and behavior of coefficients of Fourier–Vilenkin series, Positivity, 22(3), 897-918 (2018).

[2] Grigoryan M G, Sargsyan S A, On the Fourier-Vilenkin coefficients, Acta Mathematica Scientia, 37B(2), 293-300 (2017).


## Numerical Study of Mathematical Models Using Wavelet Methods


F. Shah

(University of Kashmir, India)

fashah79@gmail.com


A model is an abstraction of reality or a representation of a real object or situation. An accurate model of a process allows us to predict the process behavior for different conditions and thereby allows us to optimize and control a process for a specific purpose. Mathematical models offer an excellent methodology of conceptualizing knowledge about a process in a very compact form and are useful for formulating hypotheses, for incorporating new ideas that can later be verified in reality. Finding analytical, semi-analytical, and numerical solutions for solving mathematical models has always been an active and interesting field of research for physicists, mathematicians and biologists. Over the past decade, several numerical methods have been proposed and implemented for solving different types of mathematical models such as Adomian decomposition method, variational iteration method, differential transform method, finite difference method, operational matrix methods and *B*-spline collocation methods. Besides, some simple and accurate methods based upon orthogonal basis functions have also been developed for solving mathematical models. These methods include sinc functions, Bessel functions, block pulse functions, *B*-splines, Legendre polynomials, Jacobi and Chebyshev polynomials. Recent addition to these orthogonal functions are the wavelet functions. Wavelets are mathematical functions that detect information at different scales and at different locations throughout the computational domain. These functions can provide a bases in which the basis functions are constructed by dilating and translating a fixed function known as the *mother wavelet*. Wavelets have many excellent and attractive features: orthogonality, compact support, arbitrary regularity, and good localization. As a consequence, they are widely employed in seeking the numerical solutions of various types of differential equations, integral equations and integro-differential equations.

The present talk is devoted to a discussion of this theme to solve various types of mathematical models using wavelet collocation methods based on different types of wavelet families including Haar wavelets, Legendre wavelets, Chebyshev wavelets, CAS wavelets, Bernoulli wavelets and Gegenbauer wavelets. The performance of these numerical schemes are assessed and tested on specific test problems and the comparisons are given with other methods existing in the recent literature. The numerical outcomes indicate that these methods yields highly accurate results and are computationally more efficient than the existing ones.

## Trigonometric Polynomials and Gaussian Stationary Processes

N. Shahaf
(Georgia Institute of Technology, USA)
shahaf.nitzan@math.gatech.edu

We will discuss the relation between certain spectral properties of a Gaussian Stationary process (GSP) and the probability that this process remains positive over a long interval. In particular we will be interested in GSP's who's spectral measure has a gap around zero, and in the use of trigonometric polynomials to obtain, in this case, a sharp estimate on the above mentioned probability. This is joint work with Naomi Feldheim, Ohad Feldheim, Benjamin Jaye and Fedor Nazarov.

## The Haar System and Smoothness Spaces Built on Morrey Spaces

W. Sickel
(Friedrich-Schiller-University Jena, Germany)
winfried.sickel@uni-jena.de

For some Nikol'skij-Besov spaces $B^s_{p,q}$ the orthonormal Haar system can be used as an unconditional Schauder basis. Nowadays necessary and sufficient conditions with respect to $p, q$ and $s$ are known for this property. In recent years in a number of papers some modifications of Nikol'skij-Besov spaces based on Morrey spaces have been investigated. In my talk I will concentrate on a version called Besov-type spaces and denoted by $B^{s,\tau}_{p,q}$. It will be my aim to discuss some necessary and some sufficient conditions on the parameters $p, q, s, \tau$ such that one can characterize these classes by means of the Haar system. This is joined work with Dachun Yang and Wen Yuan (Beijing Normal University).

## On Convergence of the Fourier Series With Respect to the Multiplicative Systems

L. Simonyan
(Ayb School, Armenia)
lussimonyan@mail.ru

In this talk we will present some new results connected with convergence of Fourier double series with respect to the Vilenkin systems.

Particularly, the following theorem will be discussed:

**Theorem.** Let $\{W_k(x)\}_{k=0}^{\infty}$ be either unbounded or bounded Vilenkin system. Then, for each $0 < \varepsilon < 1$, there exist a measurable set $E \subset [0,1)^2$ of measure $|E| > 1 - \varepsilon$, and a subset of natural numbers $\Gamma$ of density 1 such that for any function $f(x,y) \in L^1(E)$ there exists a function $g(x,y) \in L^1[0,1)^2$, satisfying the following conditions:

1. $g(x,y) = f(x,y)$ on $E$,
2. the nonzero members of the sequence $\{|c_{k,s}(g)|\}$ are monotonically decreasing in all ways, where

$$c_{k,s}(g) = \int_0^1 \int_0^1 g(x,y)\overline{W_k}(x)\overline{W_s}(y)dxdy,$$

3. $\lim_{R\in\Gamma,\ R\to\infty} S_R((x,y),g) = g(x,y)$ almost everywhere on $[0,1)^2$, where $S_R((x,y),g) = \sum_{k^2+s^2\le R^2} c_{k,s}(g)W_k(x)W_s(y)$.

# References


[1] Simonyan L. S., On convergence of the Fourier double series with respect to the Vilenkin systems. Proceedings of the YSU, 2018, 52(1), p. 12-18.

[2] Grigorian M. G., Sargsyan S. A., On the Fourier-Vilenkin coefficients. Acta Mathematica Scientia, 2017, **37B**(2): 293-300.

[3] Grigorian M. G., Sargsyan S. A., On the $L_1$ convergence and behavior of coefficients of Fourier-Vilenkin series. Positivity, 2018, Vol. 22, Issue 3, pp 897-918.

# Multivariate Sampling-type Operators in Weighted $L_p$ Spaces

M. Skopina
(Saint Petersburg State University, Russia)
skopinama@gmail.com

The classical sampling expansion $\sum_{k\in\mathbb{Z}} f(M^{-j}k)\,\mathrm{sinc}(M^j x-k)$ is a special case of the quasi-projection operators (or scaling expansions)

$$Q_j(f;\widetilde{\phi},\phi)=M^j\sum_{k\in\mathbb{Z}}\langle f,\widetilde{\phi}(M^j\cdot-k)\rangle\phi(M^j\cdot-k)$$

with the Dirac delta function as $\widetilde{\phi}$ and the sinc-function as $\phi$. Another important special case is the case where $\phi$ is a band-limited function and $\widetilde{\phi}$ is locally summable. This class of quasi-projection operators includes classical Kantorovich–Kotelnikov operators, where $\widetilde{\phi}$ is the characteristic function of $[0,1]$. In this case $\langle f,\widetilde{\phi}(M^j\cdot-k)\rangle$ is the averages value of $f$ near the node $M^{-j}k$ (instead of the exact value $f(M^{-j}k)$ in the sampling expansion), which allows to deal with discontinues signals and reduce the so-called time-jitter errors. In the multidimensional case the operators $Q_j(f;\widetilde{\phi},\phi)$ are often considered with a matrix dilation $M$. Such operators are defined by the same formula, where the factor $M^j$ is replaced by $|\det M|^j$ and $\mathbb{Z}$ is replaced by $\mathbb{Z}^d$.

Approximation properties of quasi-projection operators are actively studied for different classes of functions $\phi$ and of functions/distributions $\widetilde{\phi}$. In the literature there are a lot of results providing error estimates in $L_p$-norm. Such estimates are aimed at the recovery of signals $f$, but they are not applicable to non-decaying signals and for signals whose decay is not enough to be in $L_p$, which are of interest to engineers. However such signals may belong to a weighted $L_p$ space. We consider a wide classes of quasi-projection operators with matrix dilations and band-limited $\phi$ ,and investigate their approximation order in the weighted $L_p$ spaces.

The presented results are joint results with Yu. Kolomoitsev.

## Differentiation of Generalized Bochner and Pettis Integrals and Orthogonal Series With Banach Space Valued Coefficients

V. Skvortsov

(Moscow State University, Russia )

vaskvor2000@yahoo.com

Riemann theory of trigonometric series was a starting point for investigation of interrelation between convergence or summability of orthogonal series and differentiation of certain functions associated with the series. In particular in the case of Fourier series the role of an associated function is played by the indefinite integral of the function which generates the Fourier series. Such a relation is specially tight in the case of Haar series, for which convergence of a series at a point is equivalent to differentiation in the sense of dyadic derivative of a function called quasi-measure associated with the series. Such kind of a relation gives in particular a method of solving the problem of recovering the coefficients of an orthogonal series from its sum by reducing it to the problem of recovering the primitive from its generalized derivative. This method works also in the case of Walsh, Vilenkin systems and in a more general case of system of characters of compact zero-dimensional groups.

We investigate this problem for series with Banach space valued coefficients. It is a popular theme in vector-valued Fourier analysis during the last decades to investigate if classical results about scalar-valued functions remain valid if the functions considered take values in some Banach space. Some results remain true for any Banach space. But the most frequently observed case is that it depends on the structure and geometry of the Banach spaces considered whether a result can be carried over to the vector-valued setting. A prominent example is the vector-valued extension of Carleson's celebrated theorem on point-wise convergence of Fourier series which is possible only in the case of UMD (unconditionality of martingale differences) spaces and was obtained a few years ago (see [1]) for a wide class of these spaces in the case of Fourier series with respect to Walsh and trigonometric systems. Another example is the theory of type and cotype of Banach spaces.

Here we consider the problem of recovering, by generalized Fourier formulae, the vector-valued coefficients of series with respect to classical orthogonal systems. To solve this problem some generalizations of Bochner and Pettis integrals are introduced and investigated. In the case of Walsh and Haar series a suitable integral is a dyadic version of the Henstock integral (definition of the classical Henstock integral for real-valued and for Banach space valued functions see in [2]). In the simplest case of convergence everywhere this integral solves the problem with coefficients from any Banach space. The problem can be solved also for series with respect to the system of characters of any compact zero-dimensional group (non-abelian case also can be considered). As in the real valued case, the solution is obtained by reducing the problem of recovering the coefficients to the one of recovering a primitive.

At the same time some nice properties of Fourier series in the sense of these generalized integrals remain valid only for functions with values in finite dimensional spaces. A typical example (see [3]):

**Theorem 1.** *For any infinite-dimensional Banach space there exists a function with values in this space such that its Fourier-Henstock-Haar series diverges almost everywhere.*

Once again the proof is based on the differentiation properties of the integrals involved (see [2]). Moreover, the rate of growth of the partial sums in the above theorem can be $n^{\frac{1}{2}-\varepsilon}$. And this rate of divergence is close to describing the *worst* type of behavior of those partial sums that can occur in an *arbitrary* infinite-dimensional Banach space. In fact the growth $O(n^{\frac{1}{2}})$ for Fourier-Pettis-Haar series can be achieved for no Banach space with the Orlicz property, i.e., for spaces on which the identity operator is (2,1)-summing.

# References


[1] T. P. Hytonen, M. T. Lacey, Pointwise convergence of vector-valued Fourier series, Math. Ann., 357 (2013), 1329-1361.

[2] T.P.Lukashenko, V. A. Skvortsov, A.P.Solodov, Generalized integrals (in Russian), LIBROKOM (URSS group), M., 2011.

[3] V. A. Skvortsov, Integration of Banach-valued functions and Haar series with Banach-valued coefficients, Moscow University Math. Bulletin, 72 (2017), no. 1, 24-30.

# Two-Sided Estimate for the Sum of a Sine Series With Slowly Varying Coefficients*

A. Solodov
(Moscow State University, Russia)
apsolodov@mail.ru

Let's consider the sum of a sine series $g(\mathbf{b}, x) = \sum_{k=1}^{\infty} b_k \sin kx$. It is known that the sum of a sine series with convex coefficients $\mathbf{b} = \{b_k\}_{k\in\mathbb{N}}$ is positive in the interval $x \in (0, \pi)$. To estimate values of the sum near the origin traditionally was used function introduced by Salem $v(\mathbf{b}, x) = x\sum_{k=1}^{m(x)} kb_k$, $m(x) = [\pi/x]$. If the sequence of coefficients $\mathbf{b}$ is slowly varying, then the following asymptotic formula holds:

$$\frac{2}{\pi^2} v(\mathbf{b}, x) \sim \frac{b_{m(x)}}{x}, \qquad x \to +0.$$

Aljančić, Bojanić and Tomić established [1], that for any convex slowly varying null sequence $\mathbf{b}$ the following asymptotic formula holds:

$$g(\mathbf{b}, x) \sim \frac{b_{m(x)}}{x}, \qquad x \to 0.$$

In this work we refine marked result. It is shown that the difference

$$g(\mathbf{b}, x) - \frac{b_{m(x)}}{x}$$

in order is comparable with the function

$$\sigma(\mathbf{b}, x) = x \sum_{k=1}^{m(x)} k^2 (b_k - b_{k+1}).$$

A two-sided estimate of this difference with sharp constants is obtained.

*This work was financially supported by Program of the President of Support of Leading Scientific Schools (Project NSh-6222.2018.1).

**Theorem 1.** *Let* $\mathbf{b}$ *be positive convex slowly varying null sequence. Then for some* $x_0 > 0$ *the following inequalities hold:*

$$\frac{b_{m(x)}}{x} + \frac{1}{\pi^2}\sigma(\mathbf{b}, x)\big(1 + o(1)\big) < g(\mathbf{b}, x) < \frac{b_{m(x)}}{x} + \frac{1}{2}\sigma(\mathbf{b}, x)\big(1 + o(1)\big), \; 0 < x < x_0.$$

*There are convex slowly varying null sequences* $\underline{\mathbf{b}}$, $\overline{\mathbf{b}}$, *such that*

$$\varliminf_{x\to+0}\left(g(\underline{\mathbf{b}}, x) - \frac{b_{m(x)}}{x}\right)/\sigma(\underline{\mathbf{b}}, x) = \frac{1}{\pi^2},$$
$$\varlimsup_{x\to+0}\left(g(\overline{\mathbf{b}}, x) - \frac{b_{m(x)}}{x}\right)/\sigma(\overline{\mathbf{b}}, x) = \frac{1}{2}.$$

# References


[1] S. Aljančić, R. Bojanić and M. Tomić *Sur le comportement asymptotique au voisinage de zéro des séries trigonométriques de sinus à coefficients monotones*, Acad. Serbe Sci. Publ. Inst. Math., 1956, 10, 101–120.


## Characterization of Associate Function Spaces *


V. Stepanov

(Steklov Mathematical Institute of RAS, Moscow, Russia)

stepanov@mi.ras.ru


We analyse the problem of characterization of function spaces associated to a given function spaces. The situation is rather different for an ideal and non-ideal function spaces. We provide several examples of such a characterization including the weighted Sobolev space of the first order


*This research was carried out at the Steklov Mathematical Institute of RAS and financially supported by the Russian Scientific Fund (Project 14-11-00443)

on the real line. As an important corollary this characterization provides the principle of duality allowing to reduce, for instance, the weighted inequalities to a more menageable inequalities.

## Potential of the Institute of Theoretical Mathematics and Scientific Computations (Kazakhstan) in directions and themes

N. Temirgaliyev
(Institute of Theoretical Mathematics and Scientific Computations of the L.N. Gumilyov Eurasian National University)
ntmath10@mail. ru

The talk is dedicated to the 90th anniversary of the outstanding Soviet Armenian mathematician Alexander Talalyan, whose profound scientific reports at the Men'shov-Ul'yanov seminar in Lomonosov Moscow State University I was lucky to take part since 1969.

I would like to list directions and themes of research, made in Kazakhstan from the same direct apprenticeship of Soviet Russian mathematicians P.Ul'yanov and S.Voronin (see also [1-2]):

**Direction 1.** Computational (numerical) diameter (C(N)D) as a synthesis of the known and the new in numerical analysis, which, according to K. Fletcher, "includes the formulation of the problem, the mathematical analysis, the construction of the algorithm and bringing the computerization program before it gives results".

**Theme 2.** Classes (and spaces) of functions, which, according to A.N. Kolmogorov, solves the problem of "We are many", i.e. make publications available to "Many".

**Direction 3.** A mathematical tool for direct application: algebraic number theory in combination with harmonic analysis in problems of numerical integration and the theory of random numbers.

**Direction 4.** A mathematical tool for direct application: tensor products of functionals in combination with harmonic analysis in problems of numerical analysis, recovery of functions and discretization solutions of partial differential equations by the values at points

**Direction 5.** Irregular distributions and the quasi-Monte Carlo method, according to K.Roth, are promising research areas in the mathematics and computer science of the 21st century with extensive applications

**Theme 6.** Functions recovery in the C(N)D-context.

**Theme 7.** Numerical differentiation of functions in the C(N)D-context.

**Theme 8.** Discretization of solutions of partial differential equations in the C(N)D-context

**Direction 9.** Theoretical and probabilistic approach to the problems of the Analysis: the probability measures construction on functions classes

**Theme 10.** The probability-theoretic approach to the problems of the Analysis: the errors of the methods of numerical integration "on the average"regarding to probability measures on function classes

**Theme 11.** Theoretical and probabilistic approach to the problems of the Analysis: errors of the methods of recover functions and discretization of solutions of partial differential equations "on the average"regarding to probability measures on function classes

**Direction 12.** The theory of embeddings and approximations - solved and unresolved problems (with the participation of V.I. Kolyada)

**Theme 13.** Fourier series: coefficient transformation and summation

**Direction 14.** Kolmogorov prediameter from Mirbolat Sikhov

**Theme 15.** The "Morry"theory is not "trivial generalizations by replacing the Lebesgue norm with the Morry norm"

**Direction 16.** Discrete and fast "algebraic"Fourier transformation

**Direction 17.** Generators of random numbers in the context of new formulas for discrete "algebraic"Fourier transformation. Generation of Lehmer random numbers with a maximum period according to the requirements of Koweju-McPherson and their extensive applications

**Direction 18.** "Geometry of numbers"in the context of algebraic number theory

**Direction 19.** Galerkin's method and new theoretical developments with subsequent applications in the context of the always accompanying vulnerability

**Direction 20.** C(N)D is analysis of infinitely differentiable functions in the key of model results of Yerik Nurmoldin.

Five fundamental results obtained in 2016 will be demonstrated.

# References


[1] Temirgaliyev N. Introdaction of the Editor-in-Chief of the Journal "The bulletin of the L. N. Gumilyov Eurasian National University. Mathematics. Computer science. Mechanics series" about the issue purposes and the ways of its implementation // Bulletin of the L.N. Gumilyov Eurasian National University, 2018, **122** (1), 8–69.

[2] Temirgaliyev N. A few words of memories of my friend and teacher Sergei Voronin // Bulletin of the L.N. Gumilyov Eurasian National University, 2016, **115** (6), 46–64.

## Remarks on Numerical Integration, Discrepancy and Diaphony

V. Temlyakov
(University of South Carolina, USA)
temlyak@math.sc.edu

The goal of this talk is threefold. First, we present a unified way of formulating numerical integration problems from both approximation theory and discrepancy theory. Second, we discuss the $r$-smooth $L_{\mathbf{p}}$-discrepancy. The main strategic point of this discussion is to motivate the study of discrepancy for the whole range of smoothness from $r = 1$, which corresponds to the classical setting, to arbitrarily large $r \in \mathbb{N}$. Third, we move in other direction – from classes of smoothness one to classes, for which we do not impose any smoothness assumptions. We establish that even in such a general setting with no smoothness assumptions we can guarantee some rate of decay of errors of numerical integration.

## Some Kind of Quaternionic Polynomials

N. Tophuridze
(Sokhumi State University, Georgia)
ntophuridze@sou.edu.ge

In the present note we investigate some kind of quaternionic polynomials. It is shown that the Fundamental Theorem of Algebra is not valid for some of them and for other ones zero sets consist of a finite number of points and Euclidean spheres. The corresponding examples are given.

# On the Fredholm Property of Semielliptic Operators In Anisotropic Sobolev Spaces

A. Tumanyan
(Russian-Armenian University, Armenia)
ani.tumanyan92@gmail.com

We study conditions under which differential operators, acting in anisotropic Sobolev spaces in $\mathbb{R}^n$, satisfy the Fredholm property. In this talk we give necessary and sufficient conditions for the Fredholm property of semielliptic operators with the special coefficients, acting in certain anisotropic weighted Sobolev spaces in $\mathbb{R}^n$. For the special classes of semielliptic operators it is proved that the index is equal to zero. The obtained results sharpen and extend the results described in the papers [1,2]. The Fredholm property of semielliptic operators in anisotropic spaces in $\mathbb{R}^n$ is also studied in [3,4].

## References

[1] Darbinyan A. A., Tumanyan A. G. On a priori estimates and Fredholm property of differential operators in anisotropic spaces. Journal of Contemporary Mathematical Analysis. vol. 53, no. 2 (2018), p. 61–70.

[2] Darbinyan A. A., Tumanyan A. G. On necessary and sufficient conditions for Noethericity of semielliptic operators with the special coefficients. Proceedings of Russian–Armenian University (2017) N2, p. 5–13.

[3] Demidenko G. V. Quasielliptic operators and Sobolev type equations. Siberian Mathematical Journal, vol. 50, no. 5 (2009), p. 1060–1069.

[4] Karapetyan G. A., Darbinyan A. A. Index of semielliptical operator in $\mathbb{R}^n$. Proceedings of the NAS Armenia: Mathematics, vol. 42, no. 5 (2007), p. 33–50.