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MAGNETIC FIELD OF STRANGE STARS

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The generation of a magnetic field and its distribution inside a rotating strange star are discussed. The difference between the angular velocities of the superfluid and superconducting quark core and of the normal electron plasma increases because of spin-down of the star and this leads to the generation of a magnetic field. The magnetic field distribution in a star is found for a stationary value of difference of angular velocities of these components. In all parts of the star this field is determined entirely by the total magnetic moment $\mathfrak{M}$ of the star which can vary from $10^{31}-10^{34} G \cdot cm^3$ for some models of compact stars. Also the maximum possible values of magnetic field on the surface of various models of strange dwarfs have been estimated. Depending on configuration parameters, mass M and radius R of the star, the limit of $10^3-10^5 G$ has been established. Such values of magnetic field may be an additional condition for identification of strange dwarfs among the large class of observed white dwarfs.

***Keywords*:** quark matter, strange star, superfluidity, magnetic field.

1. Introduction. It is known that a phase transition from baryonic matter to quark plasma is possible at supernuclear densities. It was shown in [1–3] that the appearance of strange quark matter with the occurrence of "*s*" quarks (the "CFL" phase) is energetically more efficient. Strange quark material can form self-sustaining bound states in the form of "strange stars" (SQS) even in the absence of gravitation. Such a body may also be the core of an ordinary neutron star and white dwarf.

Matter in the quark phase can exist in the normal state and in a superfluid and superconducting state. For temperatures $T < T_c \approx 50\, MeV$, the quark matter is in a superfluid and superconducting state. For densities $\rho \gg \rho_0$ (where ρ_0 is the nuclear saturation density), we are dealing with the "CFL" phase of quark matter [4]. The superfluid and superconducting state of the "CFL" phase consists of paired massless "*u*", "*d*" and "*s*" quarks of all three colors [5, 6]. Note that in the above model for SQS, the density of "s" quarks decreases on approaching the boundary of the quark core and this leads to the appearance of electrons, which maintain overall charge neutrality [7–9]. Since electrons are bound to the quark core only by the Coulomb interaction, they may abandon the quark surface and form an electron plasma having a thickness on the order of $10^2-10^3\, fm$. For this reason, a thin charged layer appears at the surface of a strange quark star, where the electric field intensity attains values of $10^{17}-10^{18}\, V/cm$ [10–12]. The electric field in the near-surface charged layer is directed outward. Consequently, it may support a crust that consists of atomic nuclei and degenerate electrons (the *Ae* phase), which is bound to strange quark material by gravity.

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In the present article we refine the mechanism for generation of a magnetic field in a rotating SQS taking into account the superfluidity and superconductivity of the quark material. We shall see below that this accounting leads naturally to the difference $\Delta\Omega = \Omega_s - \Omega_n$ in the angular velocities Ω_s of the superfluid quark material and Ω_n of the normal electron plasma required to generate a magnetic field. Section 2 considers the distribution of the electric field in the near-surface layer of the quark core; Section 3 discusses the possibility of differential rotation of the superfluid and normal quark star components; Section 4 finds a magnetic field distribution in a quark star in which a crust is present; and Section 5 presents several quark star models and values for their magnetic fields. The feasibility of detecting quark stars in the form of magnetars and white dwarfs is discussed.

2. Electric Field at the Surface of a Strange Quark Core. As was shown in [9], the differential rotation of the positively charged quark core and the electron layer linked to the crust results in the appearance of a surface current:

$$i = \frac{E}{8\pi^2}\left(\Omega_s - \Omega_n\right), \tag{1}$$

where E is the radial electric field at the surface of the quark core. To determine the electric field at the surface of the quark core, we first must solve the Poisson equation for electric potential. Positive charges at the quark core surface are distributed in a layer having a thickness of the order of 15 fm [13]. However, after the manner of [10], we shall consider a simple model, in which the charge of the quark core is taken to be uniformly distributed over the entire volume (the Thomas–Fermi model). Result of numerically integrating equations [14] is shown in Fig. a). Per the diagram, the width of the electron layer between the quark core and the crust of the star is of the order of $z_e \sim 10^2 - 10^3\ fm$.

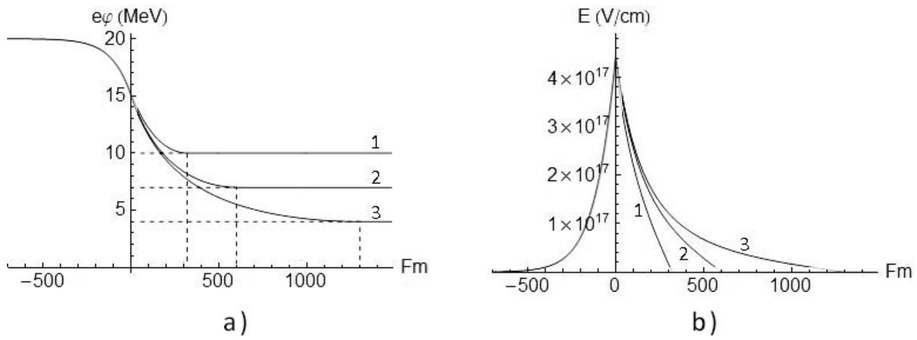

Fig. a) Potential $e\varphi(z)$; b) electric field $E(z)$ as a function of distance to the quark core for difference values of potential in the crust: 1 – $e\varphi_{cr} = 10\ MeV$; 2 – $e\varphi_{cr} = 7\ MeV$; 3 – $e\varphi_{cr} = 4\ MeV$.

Owing to the rapid change in the narrow electron layer's potential, an electric field of the order of $5\cdot 10^{17}\ V/cm$ develops in the layer (Fig. b). However, as shown in [12, 13] due to β decay of quarks near core surface the electric field can rise up to the order of $10^{18}\ V/cm$. With a value for the electric field at the surface of the quark core, we can calculate [14] the surface current density using relations (1) for a specified difference in angular velocities of the superfluid core and the normal crust $\Omega_s - \Omega_n$.

If quark matter were in a normal state, then for a specified surface current $i = \text{const}$, the magnetic field would be uniform inside the quark core and dipolar outside the core [9, 15].

However, the quark core is in a superfluid and superconducting state, which results in a change in magnetic field distribution and the appearance of differential rotation in the strange quark star.

3. Generation and Distribution of Magnetic Field in the Quark Star. In [16], based on a topological and group theoretical analysis of the free Ginzburg–Landau energy for the CFL phase, new non-Abelian superfluid and superconducting vortices M_1 were found, simultaneously exhibiting quantized mechanical moment and quantized magnetic flux, the density of which was proportional to the angular velocity of quark matter Ω_s:

$$n = \frac{2\Omega_s}{\chi}, \quad \chi = \frac{\pi\hbar}{m_B}, \tag{2}$$

where χ is the quantum of circulation for superfluid vortices M_1, and m_B is the baryon mass. From this expression, it follows that when the quark core slows down, i.e. when $\dot{\Omega}_s < 0$, the density of superfluid vortices M_1 is reduced. This means that these vortices move outward, but since they also possess magnetic moment, electron scattering will occur due to the vortex magnetic field M_1. Thus, the motions of the superfluid and normal components of a quark star are interrelated and it may be said that the motion of superfluid vortices M_1 are accompanied by friction due to the normal component of the star. Since an analogous situation occurs for neutron star rotation, then the equations of rotation dynamics for a neutron star, examined in [17], may also be used to study a two-component quark star. For the steady-state value of difference between angular velocities of superfluid and normal components $\Delta\Omega = \Omega_s - \Omega_n$ we have [17]:

$$\Delta\Omega_{st} = \Omega_n \frac{\tau_0}{\tau}, \tag{3}$$

where $\tau_0 = \dfrac{1}{2k\Omega_s}$, $\quad k = \dfrac{\chi\rho_s/\eta}{1+(\chi\rho_s/\eta)^2}$ and τ is the age of the quark star, ρ_s is the density of the superfluid matter, while η is the friction coefficient between the vortex and the normal component.

We consider a quark star of radius R, possessing a spherical core of radius a, consisting of color superconducting quark matter. The core is surrounded by a normal component consisting of an electron layer and a crust having an overall thickness equal to $R-a$. Differential rotation of the superfluid quark core and the normal component results in the generation of a magnetic field. Rotation of the quark color charge also generates a gluomagnetic field in the CFL phase of quark matter. It turns out that the electromagnetic and gluomagnetic fields are interrelated owing to the complex structure of one of the gluons, which results in so-called "rotational electromagnetism". The magnetic and gluomagnetic field may be described by vector potentials $\vec{A}(r,\vartheta)$ and $\vec{A}_{8,p.r,\vartheta}$, which are governed by the Ginzburg–Landau equations [16, 18–21]:

$$\lambda_q^2 \operatorname{rot}\operatorname{rot}\vec{A} + \sin^2\alpha\vec{A} = \vec{f}\sin\alpha + \sin\alpha\cos\alpha\vec{A}_8, \tag{4}$$

$$\lambda_q^2 \operatorname{rot}\operatorname{rot}\vec{A}_8 + \cos^2\alpha\vec{A}_8 = -\vec{f}\cos\alpha + \sin\alpha\cos\alpha\vec{A}, \tag{5}$$

where the depth of penetration λ_q and the angle of magnetic and gluomagnetic field "mixing" are defined in [21, 22]. Since quark matter in the CFL phase is a type-II superconductor, the magnetic and gluomagnetic fields may penetrate into the quark core by means of these quantum vortices. Solving the system of equations (4) and (5) for $\vec{A}(r,\vartheta)$ and $\vec{A}_{8,p,r,\vartheta}$, using the relation of magnetic field and vector potential $\vec{B} = \operatorname{rot}\vec{A}$, and taking into account the boundary conditions, we find the values of magnetic field components in the quark core, where $r < a$ [22]:

$$B_r^q = \frac{2\mathfrak{M}}{a^3}\cos\vartheta, \quad B_\vartheta^q = -\frac{2\mathfrak{M}}{a^3}\sin\vartheta, \tag{6}$$

where $\mathfrak{M}$ is the total magnetic moment of the rotating quark core:

$$\mathfrak{M} = \frac{Ea^4}{3c}\Delta\Omega. \tag{7}$$

The magnetic field outside the core, where $r > a$, is found in [21,22] and has the form:

$$B_r^n = \frac{2\mathfrak{M}}{r^3}\cos\vartheta, \quad B_\vartheta^n = \frac{\mathfrak{M}}{r^3}\sin\vartheta. \tag{8}$$

For the magnetic field on star's surface we obtain:

$$B^{ext} \sim \frac{\mathfrak{M}}{R^3} = \frac{Ea^4}{3cR^3}\Delta\Omega. \tag{9}$$

Thus, star's magnetic moment $\mathfrak{M}$ and external magnetic field B are proportional to the difference between the angular velocities of the quark matter and the electron plasma $\Delta\Omega$. When the star's rotation slows down this difference increases, since Ω_n initially decreases and then only Ω_n follow it. The increase in $\Delta\Omega$ leads to a rise in the tangential component of the magnetic field in the normal phase. When this field reaches the value of H_{c1} characterizing the lower superconducting critical field for quark matter, the vortices formed at the surface of the quark core move inward carrying magnetic flux with them.

4. Models of Strange Quark Stars and Their Magnetic Fields. Strange quark matter, is the principal state of cold matter bound by the strong interaction. Strange quark matter may be described using three phenomenological parameters: the constant B from the MIT equations of state, the quark-gluon interaction constant α_c, and the mass of the strange quark m_s. The existence of self-sustaining strange stars is possible for certain values of these parameters. The principal properties of star configurations consisting of a strange quark core and crust were studied in [8, 23].

We evaluate the value of the quark star's magnetic field B according to the formula (9). The maximum magnetic field may be obtained, if we assume that $\Delta\Omega \sim \Omega_s \sim 10^4 \text{rad}/s$, which is the maximum possible value for bare quark star [24]. For the electric field we take the value $E \sim 10^{18} V/cm \sim 3 \cdot 10^{15} CGSE$.

For large star mass ($M \sim M_\odot$), the crust thickness attains a value of the order of 5% of the overall star radius. If for the model of a quark star of mass $M/M_\odot = 1$ we take the radius of neutron star $a \sim R \sim 10$ *km*, then we obtain magnetic field at the star's surface $B^{ext} \sim 3 \cdot 10^{14}$ *G*, which is of the order of a hypothesized magnetar field.

If the mass of a quark core $M < M_\odot$, then the crust swells significantly and its maximum radius is on the order of that of white dwarfs. These configurations referred to as "strange dwarfs". We also calculate the values of magnetic fields for models of strange dwarfs given in [23]. As is seen from calculations, the maximum values of magnetic field can attain $B \sim 10^3 - 10^5$ *G*. In [25] seven objects were chosen from among white dwarfs as candidates for strange dwarf. Magnetic fields of five of these seven (GD 140, EG 50, GD 279, LTT 7987, G 238-44) were given in [26–28]. The mechanism of generation of magnetic field discussed in this work can explain observed values of magnetic fields of the objects mentioned above.

There were observed white dwarfs the values of magnetic field of which are greater than 10^5 *G* [29]. We should note, that present mechanism of magnetic field generation cannot explain so great fields of white dwarfs.

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