AN APPROACH TO PLANNING AN ADVERTISING CAMPAIGN

Mels Sahakyan
Yerevan State University, Faculty of Economy, Armenia
mels.sahakyan@gmail.com

Gayane Tumanyan
Yerevan State University, Faculty of Economy, Armenia
tum.gayane@gmail.com

ABSTRACT

An operations research approach to planning an advertising campaign of goods is employed to demonstrate the expediency of the use and the power of applied mathematics tools in solving business and trade problemsIn this work we propose a new approach to plan an advertising campaign using three models. It is shown that under certain natural assumptions, the considered planning problem can be formulated as an optimization one, namely, as an integer linear programming problem and nonlinear programming problem with linear constraints, in which the constraints represent the value of the advertising budget, total number of commercials and expected revenue., whereas the goal function is the GRPs of advertising. We also conducted comparative assessment of advertising campaign effectiviness to applying data envelopment analysis method to classify channels according to the level of efficiency.

Keywords: *advertising campaign, DEA method, expected revenue, Gross Raiting Points, Nonlinear programming*

1. INTRODUCTION

Today any organization to maintain and develop its position in the market uses advertising. In this regard, the organization's task is to correctly plan and implement advertising campaign from point of view of marketing objectives. In this work we proposed a new approach to planning an advertising campaign using three models. To plan an advertising campaign is necessary:

- Make a preliminary selection of TV channels guided by the index of audience share.
- Determine the number of commercials for each channel, in which the Gross Rating Points of advertising will be maximum, within the limits of budget and preferable expected revenue.
- Determine the best and most probable allocations of TV Commercials.
- Estimate the comparative effectiveness of the Advertising Campaign.

1.1. Explanations of terms used in this study

Advertising breaks - the airtime, during which is made in advertising.

TV channel audience - the number of people watching the channel.

TV Rating: the estimate of the size of television audience relative to the potential audience, expressed as a percentage. The estimated percent of all TV households or persons tuned to a specific station.

Share of audience (Shr) - Share is the audience of particular television program or time period expressed as a percent of the population viewing TV at that particular time.

Gross Rating Points (GRP) – A unit of measurement of audience size. It is used to measure the exposure to one or more programs or commercials, without regard to multiple exposures of the same advertising to individuals.

$$GRPs = \sum_{i=1}^{m} q_i \, n_i,$$

Where
q_i – represents the TV ratings for the time period
n_i – represents the number of broadcast commercials

2. COMMERCIALS ALLOCATION PROBLEM.

2.1 Problem description.

The company would like to advertising their products through the m different channels ($TV_i \leftrightarrow i$, i=1, 2, …, m) and get the maximum GRP of advertising within the limits of C budget allocated for advertising, at the same time we should assure.

- some preferable expected revenue
- a certain amount of commercials for advertising and so on.

2.1.1 Mathematical model.

To solve this problem we apply the integer linear programming methods. The mathematical model can be formulated as follows:
Objective function: maximization of the Gross rating points of advertising.

$$\text{GRPs} = \sum_{i=1}^{m} q_i n_i \rightarrow \max \qquad (1)$$

Constraints below:

1. limitation on total number of commercials
$$\sum_{i=1}^{m} n_i = N \qquad (2)$$
2. limitation on advertising budget
$$\sum_{i=1}^{m} d_i n_i * t/60 \le C \qquad (3)$$
3. limitation on preferable expected revenue
$$\sum_{i=1}^{m} r_i n_i * t/60 \ge R \qquad (4)$$
4. limitation on variables
$$n_i \ge 0,\ n_i \in (Z),\ (i=1,2,\ldots,m): \qquad (5)$$

where:
q_i - represents the TV ratings for the time period
d_i - The cost per minute of advertising /AMD/
r_i – The preferable expected revenue from per minutes of advertising /AMD/
t – Length (in unit) of commercial
As the advertising prices are expressed at 1 minute's value, then in constraints (3) and (4) the number of commercials turned into a minute: $n_i * \frac{t}{60}$

3. THE MOST PROBABLE CHOICE OF COMMERCIALS ACCORDING TO TV CHANNELS

We remind that the values of any linear programming problem parameters should be exact. This obligatory condition does not act for parameters of integer linear programming problem when we chose above mentioned commercial, as mainly they have been determined based on the statistical data, which means that they are not determined exact values and they contain uncertainty. Therefore, the group of feasible solutions of the problem has probabilistic characteristic. For solving the optimization problems in the uncertainty conditions, using the Entropy maximization principle, which is represented below:

3.1 Problem description.

Find the most probable allocation of commercials according to the channels by providing:

- The best value of GRP derived from commercials allocation problem
- Advertising budget
- Some preferable expected revenue

3.1.2 Mathematical model.

Find the most probable allocation of commercials applying the maximum entropy principle.
The mathematical model can be formulated as follows:
Find the most probable allocation of commercials through the channels:

$$H(n) = -\sum_{i=1}^{m} n_i \ln n_i \rightarrow max \quad (6)$$

The following conditions:

1. Assure (1) - (5) problems' objective function - GRP best value
   $$\sum_{i=1}^{m} q_i n_i \geq GRPs^{**} \quad (7)$$

Where GRPs** corresponds to the value of GRP derived from the solution of integer linear programming problem of commercials allocation.

2. Assure the total number of commercials.
   $$\sum_{i=1}^{m} n_i = N \quad (8)$$
3. Satisfy the constraint of advertising budget
   $$\sum_{i=1}^{m} d_i n_i * t/60 \leq C \quad (9)$$
4. Assure preferable expected revenue
   $$\sum_{i=1}^{m} r_i n_i * t/60 \geq R \quad (10)$$
5. Limitation on variables
   $$n_i \geq 0,\ n_i \in (Z),\ (i=1,2,\ldots,m): \quad (11)$$

4. ASSESSMENT OF ADVERTISING CAMPAIGN EFFECTIVINESS.

We conducted comparative assessment of advertising campaign effectiviness to applying data envelopment analysis method. The budget allocated to each channel we received as an input, and the GRP and the preferable expected revenue as an output.
Are considered the n decision-making units. During the period the j-th (j=1,2,…,n) decision-making unit to use the number of input x_{ij} type of i-th (i=1,2,…,m) and issued the number of output y_{rj} type of i-th r-th $(r=1,\ldots,l)$ and ($x_{ij} > 0,\ y_{rj} > 0$).
In CCR models for each j- th (j=1,2,…,n) DMU builds on weighted sum of outputs (-$\sum_{r=1}^{l} u_r y_{rj}$) and weighted sum of inputs -$\sum_{i=1}^{m} v_i x_{ij}$
Where u_r $(r=1,\ldots,l)$ and v_i $(i=1,\ldots,m)$ weighting multipliers are to be determined.
<u>In the Resource-oriented model</u> comparative efficiency for each j-th DMU is estimated in the following relation:

$$h_j(u,v) = \frac{\sum_{r=1}^{l} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$$

which is the quantity of generalized "weighted output" issued by the "weighted resource" from j-th DMU.

The Comparative effectiveness assessment for j_o DMU activity is formulated as:

$$h_{j_0}(u,v) = \frac{\sum_{r=1}^{l} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}} \to \max$$

$$\frac{\sum_{r=1}^{l} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \quad j=1,\ldots,n, \qquad (12)$$

$$u_r, v_i \ge 0,\ r=1,\ldots,l;\ i=1,\ldots,m$$

This model can be converted into a linear programming model. With some modification the (12) problem is brought to the following linear programming problem.

$$z^* = \max z = \sum_{r=1}^{l} u_r^* y_{rj_0}$$

$$\sum_{r=1}^{l} u_r^* y_{rj_0} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j=1,\ldots,n \qquad (13)$$

$$\sum_{i=1}^{m} v_i x_{ij_0} = 1$$

$$u_r, v_i \ge 0,\ r=1,\ldots,l;\ i=1,\ldots,m$$

The dual model of Resource-oriented problem (Farrell's model). It is a dual problem of (13) linear programming problem expressed vector form.

$$\min_{\theta,\lambda} \theta$$

$$-y_i + Y\lambda \ge 0 \qquad (14)$$

$$\theta x_i - X\lambda \ge 0$$

$$\lambda \ge 0$$

5. TASKS AND PRACTICAL APPLICATION OF RESULTS

The models are presented on two practical examples. The calculations has been done based on data's given by "SHARK" LLC for the purpose to execute advertising campaign for "Lek a Sandoz" Company.

5.1 Practical Example N1:

5.1.1 Calculation and result of commercials allocation problem for the example N1:

Problem formulation. Company in order to advertise its products applied to two channels. Data necessary to solve the problem are shown in table 1.

Table 1. Initial Data of practical example N1 *(Continues on the next page)*

1	Data	TV1	TV 2
		indexes ratios	indexes ratios
2	18:00 - 24:00 airtime raiting	q_1=7.4	q_2 = 0.5
3	The cost per minute of advertising - d, /AMD/	d_1=120000	d_2=24000
4	The expected revenue from per minutes of advertising - r /AMD/	r_1=200000	r_2= 60000
5	advertising budget - C	5100000	
6	total number of commercials- N, /minutes/	110	

7	the expected revenue- R, /AMD/	9500000
8	Length (in unit) of commercial /sec/	t = 60

The mathematical model can be formulated as follows:
Objective function is a maximization of the Gross rating points of advertising.

GRPs = $q_1n_1 + q_2n_2 \rightarrow$max (15)

Constraints below:

1. Limitation on total number of commercials.

$$n_1 + n_2 = N \quad (16)$$

2. Limitation on advertising budget.

$$d_1n_1 * \frac{t}{60} + d_2n_2 * \frac{t}{60} \leq C \quad (17)$$

3. Limitation on preferable expected revenue.

$$r_1n_1 * \frac{t}{60} + r_2n_2 * \frac{t}{60} \geq R \quad (18)$$

4. Limitation on variables.

$n_1, n_2, \geq 0, n_i \in (Z)$, (i=1,2,…,m): (19)

To solve (15)- (19), we get n_1= 25, n_2= 85; GRPs = 227.5, C=5040000, R=10100000;
Therefore in the N1 TV channel should provide 25 spot, N2 TV channels should provide 85 spot and maximum Gross Rating Points is GRPs=227.5.
We have conducted calculations change of the commercials total number in order to find the maximum budget.
Calculation results for modifications in total number of commercials /the other conditions are not modified /are presented in table 2/:
In particular consider

- 7-th case when $\sum n_i \leq 110$:

The results are as follows: n_1=32, n_2= 52; GRPs = 262.8, C=5088000**,** $N = 84$:

- 8-th case when $\sum n_i \geq 110$:

The results are as follows: n_1= 25, n_2= 87; GRPs = 228.5, C=5088000**,** $N = 112$ **:**

Table 2: Calculations results of changes of the total commercials number

	# **commercials**	**n_1**	**n_2**	**GRPs**
1	104	24	80	217.6
2	109	25	84	227
3	***110***	***25***	***85***	***227.5***
4	111	25	86	228
5	112	25	87	228.5
6	113	24	89	222.1
7	$\sum n_i \leq 110$	32	52	262.8
8	$\sum n_i \geq 110$	25	87	228.5

As we see on both cases the maximum budget that can be spent in case of integer values is 5,088,000 AMD. Compared with initial budget it remains 12,000 AMD which is not enough for additional results. We see that in the 7-th the GRP is greater, than the 3-rd case, then the question arises, why chosen 3-th case, if in this case GRP is less by 35.3 unit. Selection explanation lies in the fact that the high rate of GRPs not always ensures the full effectiveness of the Advertising Campaign. There are other factors, one of which the daily frequency of

broadcast commercials /to remain in memory the average frequency of screening is considered to be 3/. In Table 2 shows that the commercials difference between 3-th and 7-th cases of 26 ad spots. The practical example N1 is represented as linear programming problem and to build an appropriate dual problem for estimate the resources costs. Let's solve (15) – (13), but we do not consider that variables n_1, n_2 are integers.
The results of linear programming problem for examples N1. n_1=25.625, n_2=84.375; GRPs=231.8, C=5100000, R=10187500.
The results of dual problem of LP for examples N1. y_1 = 1.225, y_2 = 0,0000718, y_3 = 0:
where y_1, and y_2, y_3 are estimates which correspond to respectively to the shadow prices of resources (16), (17) and (18). They estimate the lack of resources, also shows by how much the objective function will be changes if the resources increased by 1 unit. As we see

- y_1=1.225, it means if we reduce the objective function /GRP/ is 1.225, then the resource corresponding to y_1 /the total number of commercials/ will increase by 1 unit. In this case will be placed 111 commercials.
- y_3=0, it means that the resource (preferable expected revenue) corresponding to (18) limitation is not small, it is excessive /R=10187500>9500000/: It is additional benefit for the company.
- y_2 very close to zero, so the changes of the GRPs will be very little.

5.1.2 Calculation and result of the most probable allocation problem of commercials for the example N1

Solve the problem (15) - (19) for the example N1 applying the maximum entropy principle. The mathematical model can be formulated as follows:
Find the most probable allocation of commercials through the channels:

$$H(n) = - \sum_{i=1}^{m} n_i \, ln n_i \rightarrow max \quad (20)$$

The following conditions:
1. Assure (15) - (19) problems' objective function - GRP best value

$$q_1 n_1 + q_2 n_2 \geq GRPs^{**} \quad (21)$$

Where GRPs** corresponds to the value of GRP derived from the solution of integer linear programming problem of commercials allocation.
2. Assure the total number of commercials.

$$\sum_{i=1}^{m} n_i = N \Rightarrow n_1 + n_2 = N \quad (22)$$

3. Satisfy the constraint of advertising budget.

$$\sum_{i=1}^{m} d_i \, n_i * t/60 \leq C \Rightarrow \quad d_1 n_1 * \frac{t}{60} + d_2 n_2 * \frac{t}{60} \leq C \quad (23)$$

4. Assure preferable expected revenue.

$$\sum_{i=1}^{m} r_i \, n_i * t/60 \geq R \Rightarrow \quad r_1 n_1 * \frac{t}{60} + r_2 n_2 * \frac{t}{60} \geq R \quad (24)$$

5. Limitation on variables.

$$n_1, n_2, \geq 0, n_i \in (Z), \ (i=1,2,\ldots,m): \quad (25)$$

To solving the nonlinear programming problem of (20-25) /see data in the table 1/, we get` n_1=25; n_2=85; C=5040000, R=10100000; the (21) condition turned into equation and GRPs = 227,5:
As we see the allocation results of advertising commercials matches with integer linear programming results of the example N1. So, we see that the solutions of problems' integer linear programming and nonlinear programming correspond, when we apply the entropy maximum principal. Hence the solutions are also endowed with the ability to be the most probable.

5.2 Practical example N2:
5.2.1 Calculation and result of commercials allocation problem for the example N2:

№	data	TV chanells:		
		i =1	i =2	i =3
		Variables: n		
		n_1	n_2	n_3
2	airtime raiting	q_1=0.3	q_2=1.8	q_3=6,8
3	The cost per minute of advertising - d, /AMD/	d_1=24000	d_2=68000	d_3=120000
4	The expected revenue from per minutes of advertising - r /AMD/	r_1≥60000	r_2≥115000	r_3≥200000
5	advertising budget - C	9600000		
6	the expected revenue- R, /AMD/	17000000		
7	Advertising budget dedicated the third TV chanells should be no more than 65% of the total.			
8	Length (in unit) of commercial /sec/` t= 25 sec.			

Problem formulation**.** The company is ready to advertising his product on the three channels /first i=1, second i=2, third i=3/ It is necessary, within limits of the advertising budget, to get

- the maximum of advertising GRP,
- some preferable expected revenue,
- assure limitation of advertising budget allocated to 3rd channel (presented in percentages).

Data necessary to solve the problem are shown in table 3. The mathematical model can be formulated as follows:
Objective function: maximization of the Gross rating points of advertising.

$$\mathrm{GRPs} = q_1 n_1 + q_2 n_2 + q_3 n_3 \rightarrow \max \quad (26)$$

Constraints below:

1. Advertising budget dedicated the third TV chanells should be no more than 65% of the total.

$$d_3 n_3 * t/60 \leq (d_1 n_1 * t/60 + d_2 n_2 * t/60 + d_3 n_3 * t/60)0.65 \quad (27)$$

2. Limitation on advertising budget.

$$d_1 n_1 * t/60 + d_2 n_2 * t/60 + d_3 n_3 * t/60 \leq C \quad (28)$$

3. Limitation on preferable expected revenue.

$$r_1 n_1 * t/60 + r_2 n_2 * t/60 + r_3 n_3 * t/60 \geq R \quad (29)$$

4. Limitation on variables.

$n_1, n_2, \geq 0, n_i \in (Z)$, (i=1,2,…,m):
(30)

Table 3: Initial data of the example N2

To solve (26)- (30), we get n_1=117, n_2 =79, n_3=124, GRPs=1020.5 , C=9592960, R=17016480:
Example N2 is represented as linear programming problem and to build an appropriate dual problem for estimate the resources costs. Let's solve (26) – (30), but we do not consider that variables n_1, n_2, n_3 are integers.
The results of linear programming problem for examples N2.
n_1 =113.6363, n_2 =78.6703, n_3 =125,0006, GRPs = 1025.7; C=9600000, R=170000000:
The results of dual problem of LP for the examples N2.
y_1=0.0000715, y_2=0.00018, y_3=0.0000415:

y_1, y_2, y_3 correspond to shadow prices of to (27), (28) and (29) limitations resources. Because y_1, y_2, y_3 values are very close to zero, it means that, if we increase appropriate resources the effectiveness will be slight.
Continuing research we tried to find out of the selected channels which are not effective. Clients usually avoid advertising on the channel with low ratings, as the low rating does not justify spending. In our case, we have a channel with a low rating /see first TV q_1=0.3/ .let's try to find out the effectiveness of advertising on this channel or not, and how much will GRP, if we ignore the /27/ condition.
Let's solve (26. 28--30) problem. We get n_1 = 126, n_2 =0, n_3=167, GRPs= 1173.4;
In this case the GRPs higher than 1020.5, However in the 2nd TV channel advertisement is missing /n_2 =0/ and we see that in this advertising campaign for advertising is not efficient not in the 1st channel (where q_1=0.3) but in the 2nd channel where q_2=1.8/.
We can conclude that the 2nd TV channel, in the frame of its rating, has quite high price of advertisement in contrast to the 1st channel. So, the opinion that there is no sense to advertise in channel with low rating is not so true. In our case the channels have been chosen for provision of different segment audience at the expense of rating.

5.2.2 Calculation and result of the most probable allocation problem of commercials for the practical example N2

Solve the problem (26) - (30) applying the maximum entropy principle. The mathematical model can be formulated as follows:
Find the most probable allocation of commercials through the channels:

$$H(n) = - \sum_{i=1}^{m} n_i \ln n_i \rightarrow max \quad (31)$$

The following conditions:
1. Assure (26) - (30) problems' objective function - GRP best value

$$q_1 n_1 + q_2 n_2 + q_3 n_3 \geq GRPs ** \quad (32)$$

Where GRPs** corresponds to the value of GRP derived from the solution of integer linear programming problem of commercials allocation.
2. The advertising budget dedicated the third TV chanells should be no more than 65% of the total.

$$d_3 n_3 * 25/60 \leq (d_1 n_1 * 25/60 + d_2 n_2 * 25/60 + d_3 n_3 * 25/60)0.65 \quad (33)$$

3. Satisfy the constraint of advertising budget.

$$\sum_{i=1}^{m} d_i n_i * t/60 \leq C \Rightarrow d_1 n_1 * t/60 + d_2 n_2 * t/60 + d_3 n_3 * t/60 \leq C$$

(34)
4. Assure preferable expected revenue.

$$\sum_{i=1}^{m} r_i n_i * t/60 \geq R \Rightarrow r_1 n_1 * t/60 + r_2 n_2 * t/60 + r_3 n_3 * t/60 \geq R$$

(35)
5. Limitation on variables.

$n_1, n_2, n_3, \geq 0, n_i \in (Z)$, (i=1,2,…,m):

(36)
To solve the nonlinear programming problem of (31-36) /see data in the table 3/,
we get ` n_1=117, n_2 =79, n_3 = 124, GRPs = 1020.5 , C=9592960, R=17016480:
the (32) condition turned into equation and GRPs = 1020.5:
As we see the allocation results of advertising commercials matches with integer linear programming results of the example N2.
So, we see that the solutions of problems' integer linear programming and nonlinear programming correspond, when we apply the entropy maximum principal. Therefore we come to a very interesting conclusion which is the solutions of practical example N1 and N2 has also the character of being the most probable. We mention also that such result has been obtained in case of solutions of matrix games[3].

5.2.3 Assessment Results of Effectiveness Advertising Campaign for the example N2

To assessment the effectiveness of advertising campaign we apply DEA method. The budget allocated to each channel we received as a input, and the GRP and the preferable expected revenue as a output.

Table 4. Comparative Effectiveness for Practical Example N2.

TV	Advertising budget (imput)	GRPs (output)	preferable expected revenue (output)	comparative effectiveness (**θ**)
1	1168128	35.1	2920320	1
2	2234752	142.2	3779360	0.8
3	6190080	843.2	10316800	1

The problem has been solved input orientation. It follows, that to advertise on the first and third channels are more effective /**θ**=1/. Advertising effectiveness is evident on the third channel, because the results obtained for the costs are quite high. Advertising campaign effectiveness on the first channels is due to the small expenses. Advertising campaign on the second channel is ineffective, because obtained GRPs and preferable expected revenue are not so great in comparison with expenses.

6. CONSLUSION

- During the problems of advertisement management the allocation of advertising commercials according to channels can be executed in the way that the GRP will be the maximum but in the frame of the minimal advertising budget and the maximum expected revenue. Moreover, there is a connection between the the distribution of commercials, GRPs, costs and expected revenue, and if we change of one point for each resource will be another indicator of the GRPs.
- For determination of the most probable choice of advertising commercials according to TV channels, we have proposed and applied the principle of the maximum of entropy. We have obtained that the solutions of the problems' integer linear programming and nonlinear programming correspond. Therefore we come to a very interesting conclusion which is the solutions of practical example 1 and 2 have also the character of being the most probable.
- Using the DEA method we can classify channels according to the level of efficiency

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