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Research Article

Relations between the Proof Complexity Characteristics in Two Universal Proof Systems for All Variants of Many-Valued Logics

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Abstract: Formerly we have described two types of universal propositional proof systems for each variant of propositional many-valued logic. The first of introduced systems is a Gentzen-like system, the second one is based on the generalization of the notion of determinative disjunctive normal form, defined by first coauthor for two-valued logic. The last type proof systems are "weak" ones with a "simple strategist" of proof search and we have investigated the quantitative properties, related to proof complexity characteristics in them. In particular, for some class of many-valued tautologies simultaneously optimal bounds (asymptotically the same upper and lower bounds) for each of main proof complexity characteristics are obtained in the second-type systems, considered for some versions of many-valued logic. Now we investigate the relations between the main proof complexty measures in both universal systems and prove the symilar results in Gentzen-like system for the same and for other classes of many-valued tautologies as well.
Keywords: many-valued propositional logic, Gentzen-like system, determinative conjunct, determinative disjunctive normal form, elimination rule, proof complexity.

INTRODUCTION

Many-valued logic (MVL), which was created and developed in 1920 first by Łukasiewicz (Lukasiewicz, J. 1920), has in the mean time many interesting applications in such fields as Logic, Mathematics, Formal Verification, Artificial Intelligence, Operations Research, Computational Biology, Cryptography, Data Mining, Machine Learning, Hardware Design etc., therefore the investigations in area of MVL are very actual.

Two types of universal propositional proof systems are described in (Chubaryan, A., & Khamisyan, A. 2018) such that propositional proof system for every version of MVL can be presented in both of described forms. The first of introduced systems is a Gentzen-like system, the second one is based on the generalization of the notion of determinative disjunctive normal forms, defined in (Chubaryan, A. 2002) by first coauthor. The last type proof systems are "weak" ones with a "simple strategist" of proof search and we have investigated the quantitative properties, related to proof complexity characteristics in them. In particular, for some class of many-valued tautologies simultaneously optimal bounds (asymptotically the same upper and lower bounds for each proof complexity characteristics: length, size, space and width) are obtained in the second-type systems, considered for some versions of many-valued logic. In this paper we investigate the relations between the main proof complexty measures in both mentioned universal systems and prove the symilar results in Gentzen-like system for the same and for other classes of many-valued tautologies as well.

This article consists from follow main sections: Introduction, Preliminaries, in which the main notions, materials and methods are given, Main Results, in which we describe the methods of transformation of proof, given in one of mentioned systems, into some proof in the other system and give the results of proofs complexity measures comparison in these systems. In the end of paper we give Conclusion.

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2. Preliminaries.

2.1Main notions of k-valued logic.

Let E_k be the set $\left\{0, \frac{1}{k-1}, \ldots, \frac{k-2}{k-1}, 1\right\}$. We use the well-known notions of propositional formula, which defined as usual from propositional variables with values from E_k, (may be also propositional constants), parentheses (,), and logical connectives $\&, \vee, \supset, \neg$, every of which can be defined by different mode. Additionaly we use two modes of exponential function p^{σ} and introduce the additional notion of formula: for every formulas A and B the expression $\boldsymbol{A^B}$ (for both modes) is formula also.

In the considered logics either only **1** or every of values $\frac{1}{2} \leq \frac{i}{k-1} \leq 1$ can be fixed as

2.2Designated values.

Definitions of main logical functions are:

$\boldsymbol{p} \vee \boldsymbol{q} = max(p, q)$ (1) disjunction or $\boldsymbol{p} \vee \boldsymbol{q} = [(k-1)(p+q)](mod\ k)/(k-1)$ (2) disjunction,

$\boldsymbol{p}\&q = min(p, q)$ (1) conjunction or $\boldsymbol{p}\&q = \max(\boldsymbol{p} + q - 1, 0)$ (2) conjunction.

Sometimes (1) conjunction is denoted by $\wedge$.

For implication we have two following versions:

$$\boldsymbol{p} \supset \boldsymbol{q} = \begin{cases} 1, & for\ \ p \leq q \\ 1 - p + q, & for\ \ p > q \end{cases}$$ (1) Łukasiewicz's implication or

$$\mathbf{p} \supset \boldsymbol{q} = \begin{cases} 1, & for \quad p \leq q \\ q, & for \quad p > q \end{cases}$$ (2) Gödel's implication

And for negation two versions also:

$\neg\boldsymbol{p} = 1 - p$ (1) Łukasiewicz's negation or

$\neg\boldsymbol{p} = ((k-1)p + 1)(mod\ k)/(k-1)$ (2) cyclically permuting negation.

Sometimes we can use the notation $\bar{\boldsymbol{p}}$ instead of $\neg\boldsymbol{p}$.

For propositional variable $\boldsymbol{p}$ and $\boldsymbol{\delta} = \frac{i}{k-1} (0 \leq i \leq k-1)$ we define additionally "exponent" functions:

$\boldsymbol{p^{\delta}}$ as $(p \supset \delta)\&(\delta \supset p)$ with (1) implication (1) exponent,

$\boldsymbol{p^{\delta}}$ as p with $(k\text{-}1)(1-\delta)$ (2) negations. (2) exponent.

Note, that both (1) exponent and (2) exponent are no new logical functions.

If we fix **"1"** (every of values $\frac{1}{2} \leq \frac{i}{k-1} \leq 1$) as designated value, so a formula φ with variables $p_1, p_2, \ldots p_n$ is called ***1-k*-tautology** (***≥1/2-k-tautology***) if for every $\tilde{\delta} = (\delta_1, \delta_2, \ldots, \delta_n) \in E_k^n$ assigning δ_j $(1 \leq j \leq n)$ to each p_j gives the value 1 (or some value $\frac{1}{2} \leq \frac{i}{k-1} \leq 1$) of φ.

Sometimes we call ***1-k*-tautology** or ***≥1/2-k-tautology*** simply ***k*-tautology.**

Determinative Disjunctive Normal Form for MVL

The notions of determinative conjunct and determinative disjunctive normal forms are introduced at first in (Chubaryan, A. 2002) and then are described in more detail in (Aleksanyan, S. R., & Chubaryan, A. A. 2009). The notions of determinative conjunct and determinative disjunctive normal form are generalized for all variants of MVL in **(**Chubaryan, A., & Khamisyan, A. 2018**).**

For every propositional variable p in k-valued logic $p^0, p^{1/k-1}, \ldots, p^{k-2/k-1}$ and p^1 in sense of both exponent modes are the ***literals***. The conjunct K (term) can be represented simply as a set of literals (no conjunct contains a variable with different measures of exponents simultaneously), and DNF can be represented as a set of conjuncts.

Replacement-rule are each of the following trivial identities for a propositional formula $\boldsymbol{\psi}$:

for both conjunction and (1) disjunction

$\varphi\&0 = 0\&\varphi = 0, \quad \varphi \vee 0 = 0 \vee \varphi = \varphi, \quad \varphi\&1 = 1\&\varphi = \varphi, \quad \varphi \vee 1 = 1 \vee \varphi = 1,$

for (2) disjunction $\left(\varphi \vee \frac{i}{k-1}\right) = \left(\frac{i}{k-1} \vee \varphi\right) = \overbrace{\neg\neg \ldots \neg}^{i} \varphi \qquad (0 \leq i \leq k-1),$

for (1) implication

$\varphi \supset 0 = \bar{\varphi}$ with (1) negation, $\quad 0 \supset \varphi = 1, \quad \varphi \supset 1 = 1, \quad 1 \supset \varphi = \varphi,$

for (2) implication

$\varphi \supset 1 = 1$, $0 \supset \varphi = 1$, $\varphi \supset 0 = \overline{sg}\varphi$, where $\overline{sg}\varphi$ *is* 0 for φ>0 and 1 for φ=0,
for (1) negation ¬(i/k-1)=1-i/k-1 (0≤i≤k-1), $\neg\boldsymbol{\psi} = \boldsymbol{\psi}$,

for (2) negation ¬(i/k-1)=i+1/k-1 (0≤i≤k-2), $\neg\mathbf{1} = \mathbf{0}$, $\overbrace{\neg\neg \dots \neg}^{k}\boldsymbol{\psi} = \boldsymbol{\psi}$.

Application of a replacement-rule to some word consists in replacing of its subwords, having the form of the left-hand side of one of the above identities, by the corresponding right-hand side.

In (Chubaryan, A & Khamisyan, A. 2018) the following ***auxiliary relations for replacement*** are introduced as well:

for both variants of conjunction $\left(\varphi\&\frac{i}{k-1}\right) = \left(\frac{i}{k-1}\&\varphi\right) \leq \frac{i}{k-1}$ $(1 \leq i \leq k-2)$,

for (1) implication $\left(\varphi \supset \frac{i}{k-1}\right) \geq \frac{i}{k-1}$ and $\left(\frac{i}{k-1} \supset \varphi\right) \geq \frac{k-(i+1)}{k-1}$ $(1 \leq i \leq k-2)$,

for (2) implication $\left(\varphi \supset \frac{i}{k-1}\right) \geq \frac{i}{k-1}$ $(1 \leq i \leq k-2)$, $\left(\frac{i}{k-1} \supset \varphi\right) \geq \varphi$ $(1 \leq i \leq k-1)$.

Let φ be a propositional formula of k-valued logic, $P = \{p_1, p_2, \dots, p_n\}$ be the set of all variables of φ and $P' = \{p_{i_1}, p_{i_2}, \dots, p_{i_m}\}$ $(1 \leq m \leq n)$ be some subset of P.

Definition 2.2.1:

Given $\tilde{\sigma} = (\sigma_1, \sigma_2, \dots, \sigma_m) \in E_k^m$, the conjunct $K^\sigma = \{p_{i_1}{}^{\sigma_1}, p_{i_2}{}^{\sigma_2}, \dots, p_{i_m}{}^{\sigma_m}\}$ is called $\varphi - \frac{i}{k-1}$-determinative $(0 \leq i \leq k-1)$, if assigning σ_j $(1 \leq j \leq m)$ to each p_{i_j} and successively using replacement-rules and, if it is necessary, the auxiliary relations for replacement also, we obtain the value $\frac{i}{k-1}$ of φ independently of the values of the remaining variables.

Every $\varphi - \frac{i}{k-1}$ −determinative conjunct is called also φ-determinative or determinative for φ.

Definition 2.2.2.

A DNF $D = \{K_1, K_2, \dots, K_j\}$ is called determinative DNF (DDNF) for φ if $\varphi = D$ and if "1" (every of values $\frac{1}{2} \leq \frac{i}{k-1} \leq 1$) is (are) fixed as designated value, then every conjunct $K_i (1 \leq i \leq j)$ is 1-determinative ($\frac{i}{k-1}$ − determinative from indicated intervale) for φ.

Remark 2.2.

As in (Chubaryan, A. 2002) it is also easily proved, that

- if for some k-tautology φ, the minimal number of literals, containing in φ-determinative conjunct, is m,then φ-determinative DNF has at least k^m conjuncts;
- if for some k-tautology φ there is such m that every conjunct with m literals is φ-determinative, then there is φ-determinative DNF with no more than k^m conjuncts.

2.3. Definitions of universal systems for MVL .

Here we give the definitions of two universal systems, which are described in (Chubaryan, A., & Khamisyan, A. 2018).

2.3.1. The universal elimination system UE for all versions of MVL.

The axioms of Elimination systems **UE** aren't fixed, but for every formula $\boldsymbol{k - valued\ \varphi}$ each conjunct from some DDNF of $\boldsymbol{\varphi}$ can be considered as an axiom.

For k-valued logic the inference rule is *elimination rule* (ε-rule)

$$\frac{K_0 \cup \{p^0\},\ K_1 \cup \left\{p^{\frac{1}{k-1}}\right\},\ \dots,\ K_{k-2} \cup \left\{p^{\frac{k-2}{k-1}}\right\},\ K_{k-1} \cup \{p^1\}}{K_0 \cup K_1 \cup \dots \cup K_{k-2} \cup K_{k-1}},$$

where mutual supplementary literals (variables with corresponding (1) or (2) exponents) are eliminated.

Following (Chubaryan, A., & Khamisyan, A. 2018), a finite sequence of conjuncts such that every conjunct in the sequence is one of the axioms of **UE** or is inferred from earlier conjuncts in the sequence by ε-rule is called a proof in **UE**. A DNF $D = \{K_1, K_2, \dots, K_l\}$ is k-tautological if by using ε-rule can be proved the empty conjunct (∅) from the axioms $\{K_1, K_2, \dots, K_l\}$.

The completeness of these systems is obvious.

2.3.2. Sequent type system US for all versions of MVL.

Sequent system uses the denotation of sequent Γ ⊢ Δ where Γ (antecedent) and Δ (succedent) are finite (may be empty) sequences (or sets) of propositional formulas.

For every literal C and for any set of literals K the axiom sxeme of propositional system **US** is K, $C \vdash C$.

For every formula A, B, for any sets of literals K, K_i $(i = 0, \ldots, k-1)$, each σ_1, σ_2, σ from the set E_k and for $* \in \{\&, \vee, \supset\}$ the logical rules of **US** are:

$$\vdash * \frac{K \vdash A^{\sigma_1} \text{ and } \quad K \vdash B^{\sigma_2}}{K \vdash (A * B)^{\varphi_*(A,B,\sigma_1,\sigma_2)})} \qquad \vdash \exp \frac{K \vdash A^{\sigma_1} \text{ and } \quad K \vdash B^{\sigma_2}}{K \vdash (A^B)^{\varphi_{\exp}(A,B,\sigma_1,\sigma_2)})} \qquad \vdash \neg \frac{K \vdash A^{\sigma}}{K \vdash \ (\neg A)^{\varphi_\neg(A,\sigma)}}$$

$$\text{literals elimination} \vdash \frac{K_0, p^0 \vdash A, \ \ K_1, p^{\overline{k-1}}{}^{1} \vdash A, \ \ \ldots, \ \ K_{k-2}, p^{\frac{k-2}{k-1}} \vdash A, \ \ K_{k-1}, p^1 \vdash A}{K_0 \cup K_1 \cup \ldots \cup K_{k-2} \cup K_{k-1} \vdash A},$$

Where many-valued functions $\varphi_*(A, B, \sigma_1, \sigma_2)$, $\varphi_{\exp}(A, B, \sigma_1, \sigma_2)$, $\varphi_\neg(A, \sigma)$, must be defined individually for each version of MVL such, that

1) formulas $A^{\sigma_1} \supset (B^{\sigma_2} \supset (A * B)^{\varphi_*(A,B,\sigma_1,\sigma_2)})$, $A^{\sigma_1} \supset (B^{\sigma_2} \supset (A^B)^{\varphi_{\exp}(A,B,\sigma_1,\sigma_2)})$ and $A^{\sigma} \supset (\neg A)^{\varphi_\neg(A,\sigma)}$ must be k-tautology in this version,
2) if for some $\sigma_1, \sigma_2, \sigma$ the value of $\sigma_1 * \sigma_2$ $({\sigma_1}^{\sigma_2}, \neg\sigma)$ is one of ***designed values*** in this version of MVL , then $(\sigma_1 * \sigma_2)^{\varphi_*(\sigma_1,\sigma_2,\sigma_1,\sigma_2)} = \sigma_1 * \sigma_2$ $(({\sigma_1}^{\sigma_2})^{\varphi_{\exp}(\sigma_1,\sigma_2,\sigma_1,\sigma_2)} = {\sigma_1}^{\sigma_2}, (\neg\sigma)^{\varphi_\neg(\sigma,\sigma)} = \neg\sigma)$.

We use the well known notion of proof in sequent systems. We say that formula ***A is derived in US iff the sequent*** $\vdash$ ***A is deduced in US.***

Completeness of US is proved in (Chubaryan, A., & Khamisyan, A. 2018).

2.4. Definitions of main proof complexity measures.

Four main characteristics of the proof are considered in the theory of proof complexity. Following (Filmus, Y. *et al.*, 2012) we give the formal definitions of all proof complexity measures.

If a proof in the system $\boldsymbol{\Phi}$ is a sequence of lines, where each line is an axiom, or is derived from previous lines by one of a finite set of allowed inference rules, then a $\boldsymbol{\Phi}$-configuration is a set of such lines. A sequence of $\boldsymbol{\Phi}$-configurations $\{\boldsymbol{D_0}, \boldsymbol{D_1}, \ldots, \boldsymbol{D_r}\}$is said to be $\boldsymbol{\Phi}$-derivation if $\boldsymbol{D_0}$ is empty set and for all $\boldsymbol{t}$ $(\boldsymbol{1 \le t \le r})$ the set $\boldsymbol{D_t}$ is obtained from $\boldsymbol{D_{t-1}}$by one of the following derivation steps:

Axiom Download: $\boldsymbol{D_t = D_{t-1}} \cup \{\boldsymbol{L_A}\}$, where $\boldsymbol{L_A}$ is an axiom of $\boldsymbol{\Phi}$.

Inference: $\boldsymbol{D_t = D_{t-1}} \cup \{\boldsymbol{L}\}$, for some $\boldsymbol{L}$ inferred by one of the inference rules for $\boldsymbol{\Phi}$from a set of assumptions, belonging to $\boldsymbol{D_{t-1}}$.

Erasure: $\boldsymbol{D_t \subset D_{t-1}}$.

A $\boldsymbol{\Phi}$-proof of a tautology $\boldsymbol{\varphi}$ is a $\boldsymbol{\Phi}$-derivation $\{\boldsymbol{D_0}, \boldsymbol{D_1}, \ldots, \boldsymbol{D_r}\}$ such that $\widetilde{\boldsymbol{\varphi}} \in \boldsymbol{D_r}$, where $\widetilde{\boldsymbol{\varphi}}$ is empty conjunct in ***UE*** and $\tilde{\tilde{\boldsymbol{\varphi}}}$ is $\vdash \boldsymbol{\varphi}$ in ***US***.

By $|\boldsymbol{\varphi}|$ we denote the size of a formula $\boldsymbol{\varphi}$, defined as the number of all logical signs entries. It is obvious that the full size of a formula, which is understood to be the number of all symbols, is bounded by some linear function in $|\boldsymbol{\varphi}|$.

The $\boldsymbol{size(l)}$ of a $\boldsymbol{\Phi}$-derivation is a sum of the sizes of all lines in a derivation, where lines that are derived multiple times are counted without repetitions. The $\boldsymbol{steps(t)}$ of a $\boldsymbol{\Phi}$-derivation is the number of axioms downloads and inference steps in it. The $\boldsymbol{space(s)}$ of a $\boldsymbol{\Phi}$-derivation is the maximal space of a configuration in a derivation, where the space of a configuration is the total number of logical signs in a configuration, counted with repetitions. The $\boldsymbol{width(w)}$ of a $\boldsymbol{\Phi}$-derivation is the size of the widest line in a derivation.

Let $\boldsymbol{\Phi}$ be a proof system and $\boldsymbol{\varphi}$ be a tautology. As known the minimal possible value of $\boldsymbol{t-complexity}$ $(\boldsymbol{l-complexity}, \boldsymbol{s-complexity}, \boldsymbol{w-complexity})$ for all proofs of tautology $\boldsymbol{\varphi}$ in $\boldsymbol{\Phi}$is denoted by $\boldsymbol{t_\varphi^\Phi}(\boldsymbol{l_\varphi^\Phi}, \boldsymbol{s_\varphi^\Phi}, \boldsymbol{w_\varphi^\Phi})$.

Let Φ_1 and Φ_2 be two different proof systems.

Definition 2.4.1.1.

The system Φ_1 *p-simulates* the system Φ_2 if there exist the polynomial *p()* such, that for each formula φ provable both in the systems Φ_1 and Φ_2, we have $l_\varphi^{\Phi 1} \le p(l_\varphi^{\Phi 2})$.

Definition 2.4.1.2.

The systems Φ_1 and Φ_2 are *p-equivalent*, if systems Φ_1 and Φ_2 *p-l-simulate* each other.

3.Main Results.

Here we give the algorithms of proofs transformation from one system into other system and the results of proofs complexity measures comparison in these systems, which are obtained on the base of transformation algorithms.

US→ UE Algorithm.

Let A be some k-tautology and $\Gamma_1 \vdash \Delta_1, \Gamma_2 \vdash \Delta_2, \dots, \Gamma_t \vdash \Delta_t$ be some US-proof of ⊢A with minimal possible steps. Without violation of community we can change the order of proof sequents such that each use of "elimination rule" is after the all uses of " $\vdash *$ ", " $\vdash$ exp" and " $\vdash \neg$" rules. Let $\Gamma_1 \vdash \Delta_1, \Gamma_2 \vdash \Delta_2, \dots, \Gamma_r \vdash \Delta_r, \Gamma_{r+1} \vdash \Delta_{r+1}, \dots, \Gamma_t \vdash \Delta_t$ be such US-proof of ⊢A, where only sequents $\Gamma_{r+1} \vdash \Delta_{r+1}, \dots, \Gamma_t \vdash \Delta_t$ are obtained by "elimination rule". After them we choose from sequents $\Gamma_1 \vdash \Delta_1, \Gamma_2 \vdash \Delta_2, \dots, \Gamma_r \vdash \Delta_r$ only such sequents, succedent of which is A. It is obvious, that the set of literals in antecedents of such sequents are A-determinative and now we fix them as axioms in UE. Then we must use the "elimination rule" of UE in the same order, as "elimination rules" of US.

It is not difficult to see that

$$t_A^{UE} \le t - r \le t_A^{US},$$
$$l_A^{UE} \le l_A^{US}, \quad s_A^{UE} \le s_A^{US}, \quad w_A^{UE} \le w_A^{US}.$$

So, the system UE *p-simulates* the system US.

UE → US Algorithm.

Let A be some k-tautology and $K_1, K_2, \dots, K_l, K_{l+1}, \dots, K_t$ be some UE-proof of A with minimal possible steps, where the conjuncts $K_1, K_2, \dots, K_l$ are all axioms of this proof. As every conjunct K_i $(i = 1, \dots, l)$ is A-determinative, then we can derive at first all sequents $K_i \vdash A$ $(i = 1, \dots, l)$, using the rule " $\vdash *$ ", " $\vdash$ exp" and " $\vdash \neg$" step by steps to subformulas of A with some exponents (see generalization of Kalmar's proof from [2]). Then we must use the "elimination rule" of US in the same order, as "elimination rules" of UE are used in the last part $K_{l+1}, \dots, K_t$ of given UE-proof.

It is not difficult to see that

$$t_A^{US} \le l|A| + t - l \le t_A^{UE}|A|,$$
$$l_A^{US} \le t_A^{UE}|A||A| \le l_A^{UE}|A||A|,$$
$$s_A^{US} \le s_A^{UE}|A||A|, \quad w_A^{US} \le w_A^{UE}|A|.$$

Note that there are many sequences of k-tautologies A_n , sizes of which can be very long, but $t - complexities$ of their UE-proofs are bounded by some constant, therefore the system US does not *p-simulate* the system UE and the systems UE and US do not be *p-equivalent* , but nevertheless some classes of k-tautologies have the same proof complexities bounds in both systems.

Bounds of proof complexity measures of some classes of *k*-tautologies in some variants of *UE* and US.

In some papers in area of propositional proof complexity for 2-valued classical logic (Chubaryan A, 2002 & Aleksanyan, S. R *et al*, 2009) the following tautologies (Topsy-Turvy Matrix) play key role

$$TTM_{n,m} = \bigvee_{(\sigma_1,\sigma_2,\dots,\sigma_n)\in E^n} \bigwedge_{j=1}^{m} \bigvee_{i=1}^{n} p_{i_j}{}^{\sigma_j} \; (n \ge 1, 1 \le m \le 2^n - 1).$$

For all fixed $n \ge 1$ and m in above indicated intervals every formula of this kind expresses the following true statement: given a 0,1-matrix of order $n \times m$ we can "topsy-turvy" some strings (writing 0 instead of 1 and 1 instead of 0) so that each column will contain at least one 1.

In (Chubaryan, A, *et al*, 2016, 2017, 2018 & Tshitoyan, A., 2017) the notion "topsy-turvy" is generalized as follow:

Definition 3.3.

Given $\tilde{\sigma} = (\sigma_1, \sigma_2, \dots, \sigma_m) \in E_k^m$ and $\boldsymbol{\delta} = \frac{i}{k-1}$ (0≤*i*≤*k*-1) we call **δ**-(1)-topsy-turvy-result (**δ**-(2)-topsy-turvy-result) the cortege $\widetilde{\sigma\delta}$, which contains every σ_j ($1 \le j \le m$) **with (1) exponent δ for (1)**

negation (**with (2) exponent δ for *(2) negation***) .

On the base of this notion many k-tautologies were described and their proof complexities measures was investigated in UE systems for some variants of MVL in (Chubaryan, A., & Khamisyan, A. 2018; Chubaryan, A. A. *et al.*, 2016; Tshitoyan, A. 2017).

Main Theorem.

1) In US systems for MVL with (1) conjunction, (1) or (2) disjunction, (1) implication, (1) negation ((1) conjunction, (1) disjunction, (2) implication, (2) negation) for 1-k-tautologies (k≥3) $\boldsymbol{\varphi_n} = \mathbf{TTM_{n,m}}$ for every $\mathbf{n \ge 1}$ and $\mathbf{m = k^{[n/k]}}$, where $\mathbf{TTM_{n,m}} = \bigvee_{(\sigma_1,\sigma_2,\dots,\sigma_n)\in E_k^n} \bigwedge_{j=1}^{m} \bigvee_{i=1}^{n} \mathbf{p_{i_j}}^{\sigma_j}$ the following bounds are true

$log_k log_k(t(\varphi_n)) = \theta(n)$; $log_k log_k(l(\varphi_n)) = \theta(n)$;
$log_k(s(\varphi_n)) = \theta(n)$; $log_k(w(\varphi_n)) = \theta(n)$.

2) In US systems for MVL with (1) conjunction, (1) or (2) disjunction, (1) implication, (1) negation for ≥1/2-k-tautologies (k≥3) $\boldsymbol{\varphi_n}=\mathbf{LTTM_{n,m}}$, for every $\mathbf{n \geq 1}$ and and $\mathbf{m = 2^n - 1}$ where $\mathbf{LTTM_{n,m}} = \vee_{(\sigma_1,\sigma_2,\dots,\sigma_n)\in E^n} \wedge_{j=1}^{m} \vee_{i=1}^{n} \mathbf{p_{i_j}}^{\sigma_j}$, where E={0,1} , the following bounds are true

$log_2 log_k(t(\varphi_n)) = \theta(n);$ $log_2 log_k(l(\varphi_n)) = \theta(n);$
$log_2(s(\varphi_n)) = \theta(n);$ $log_2(w(\varphi_n)) = \theta(n).$

3) In US systems for MVL with (1) conjunction, (1) or (2) disjunction, (2) implication, (2) negation for ≥1/2-k-tautologies (k≥3) $\boldsymbol{\varphi_n}=\mathbf{GTTM_{n,m}}$, for every $\mathbf{n \geq 1}$ and and $\mathbf{m} = \mathrm{k}^{\mathbf{n}} - \mathbf{1}$ where $\mathrm{G}\mathbf{TTM_{n,m}} = \vee_{(\sigma_1,\sigma_2,\dots,\sigma_n)\in E_k^n} \wedge_{j=1}^{m} \vee_{i=1}^{n} \mathbf{p_{i_j}}^{\sigma_j}$ the following bounds are true

$log_k log_k(t(\varphi_n)) = \theta(n);$ $log_k log_k(l(\varphi_n)) = \theta(n);$
$log_k(s(\varphi_n)) = \theta(n);$ $log_k(w(\varphi_n)) = \theta(n).$

Proof of upper bounds is obtained from the same upper bounds, given in [6-9] for the systems UE, from the bounds, following after **UE → US** Algorithm and from equation $log_k(|\varphi_n|) = \theta(n)$ as well.

Proof of lower bounds is obtained from the same lower bounds, given in (Chubaryan, A. A. *et al.*, 2016; Tshitoyan, A. 2017) for the systems UE and from the bounds, following after **US → UE** Algorithm.

CONCLUSION

The analogous bounds of proof complexity measures can be obtained in US and UE type systems for all variants of MVL. The preference of such systems is the simple strategy of proof steps choice and the possibility of the automatic receipt of exponential lower bounds for tautologies with specific properties: minimal numbers of literals in determinative conjunct must be by order nearly equal to the size of formula.

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