

Mechanics

DIFFRACTION OF SHEAR PLANE ELECTRO-ELASTIC WAVE ON THE SEMI-INFINITE METALLIC LAYER IN THE PIEZOELECTRIC SEMI-SPACE

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The diffraction of plane shear electro-elastic wave on a thin semi-infinite  $\Phi_\infty(x, y), w_\infty(x, y)$  metallic layer piezoelectric-vacuum space is considered in the presence of a thin infinite (ground shield) metallic layer in vacuum. The problem is reduced to the solution of a functional equation in the theory of analytical functions. The problem admits closed solution describing the wave fields in vacuum and in the piezoelectric medium. The presence of the semi-infinite metallic layer leads to a diffraction of a waves, a result of which surface electro-elastic waves occur.

**Keywords:** diffraction of waves, metallic layer, piezoelectric, vacuum, factorization.

In the present paper 6 *mm* class piezoelectric medium of hexagonal symmetry is considered that occupies  $y > 0$  semi-space in the Cartesian coordinate system,

$\Phi_\infty(x, y), w_\infty(x, y)$   $y < 0$  semi-space being in vacuum and  $y = 0, x < 0$  semiplane being metalized with grounded thin layer. The layer is so thin that its stiffness is not taken to account, i.e. the layer is considered to be an electrode on the mentioned semiplane. *OZ* axis is coincident with the piezocrystal.

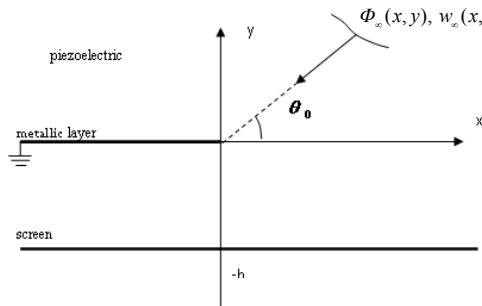


Figure.

As  $y = -h$  plane is screened, its electric potential  $\Phi^{(1)} = 0$ .

A plane shear electro-elastic wave is incident from infinity on the electrode at  $\theta_0$  angle, where  $0 < \theta_0 < \pi/2$  (Fig.). The components of electric and displacement potential amplitudes of the incident wave are as follows:

$$w_\infty(x, y) = e^{-ik_x \cos \theta_0 x - ik_y \sin \theta_0 y}, \quad \Phi_\infty(x, y) = \frac{e_{15}}{\epsilon_{11}} e^{-ik_x \cos \theta_0 x - ik_y \sin \theta_0 y}, \quad (1)$$

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where  $k = \omega / c$  is the wave number,  $c = \sqrt{c_{44}(1 + \chi) / \rho}$  is the velocity of propagating shear electro-elastic wave,  $\omega$  is the frequency of vibrations and  $\rho$  is the medium density,  $\chi = e_{15}^2 / \varepsilon_{11} c_{44}$  is the electromechanical coupling coefficient of the piezoelectric medium and  $e_{15}$ ,  $\varepsilon_{11}$ ,  $c_{44}$  are the piezoelectric, dielectric and elastic constants respectively.

Here it was taken into account that the dependence of the electro-elastic field on the time parameter is harmonic:  $e^{-i\omega t}$ , where  $t$  is the time parameter.

Subject to a condition that the piezoelectric medium is in an anti-plane deformation state, we aimed at determination of wave fields in the piezoelectric medium and vacuum. Having in view the determination of amplitudes of displacement and electric potentials, we shall avail of the following equations [1]:

$$\Delta w + k^2 w = 0, \quad \Delta \Phi + k^2 e_{15} / \varepsilon_{11} w \quad (2)$$

and for vacuum:

$$\Delta \Phi^{(1)} = 0, \quad y < 0, \quad (3)$$

where  $\Delta = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$ .

Now write out the boundary conditions of the problem under assumption of the absence of stress at the interface between piezoelectric medium and vacuum:

$$\sigma_{yz} = c_{44} \partial w / \partial y + e_{15} \partial \Phi / \partial y = 0, \quad y = 0. \quad (4)$$

The conditions for electric potential and  $D_2$  component of induction vector are:

$$y = 0, \quad x < 0, \quad \Phi = \Phi^{(1)} = 0, \quad (5)$$

$$y = 0, \quad x < 0, \quad \Phi = \Phi^{(1)} = \Phi_0(x), \quad (6)$$

$$y = -h, \quad \Phi^{(1)} = 0, \quad (7)$$

$$y = 0, \quad x < 0, \quad D_2 - D_2^{(1)} = D_0(x), \quad (8)$$

$$y = 0, \quad x > 0, \quad D_2 = D_2^{(1)}, \quad (9)$$

where  $D_2^{(1)}(x, y) = -\varepsilon_0 \partial \Phi / \partial y$ ,  $D_2(x, y) = e_{15} \partial w / \partial y + \varepsilon_{11} \partial \Phi / \partial y$ .

Hence, in the problem under consideration vacuum appears a vacuum layer of  $h$  width. Here  $\Phi_0(x)$ ,  $D_0(x)$  are the required functions, for which the following functions are applicable:

$$\Phi^+(x) = \Phi_0(x)\theta(x), \quad \Psi^-(x) = D_0(x)\theta(-x).$$

In that case from (5), (6), (8), (9) we obtain:

$$\Phi = \Phi^{(1)} = \Phi^+(x), \quad D_2 - D_2^{(1)} = \Psi^-(x),$$

where  $\theta(x)$  is the Heaviside function.

Then introducing the following functions

$$u(x, y) = w(x, y) - w_\infty(x, y), \quad \varphi(x, y) = \Phi(x, y) - \Phi_\infty(x, y),$$

we have for stress and induction components:

$$\sigma_{yz} = c_{44} \partial u / \partial y + e_{15} \partial \varphi / \partial y - ikc_{44}(1 + \chi) \sin \theta_0 e^{-ikx \cos \theta_0 - ik_y \sin \theta_0},$$

$$D_2(x, y) = e_{15} \partial u / \partial y - \varepsilon_{11} \partial \varphi / \partial y.$$

To determine the functions  $u(x, y)$ ,  $w(x, y)$ , we have from equation (2) that must also satisfy the outgoing wave condition

$$\Delta u + k^2 u = 0, \quad \Delta \varphi + k^2 \varepsilon_{15} / \varepsilon_{11} u = 0, \quad y > 0. \quad (10)$$

Applying the generalized integral Fourier transform [2], we obtain

$$\frac{d^2 \bar{u}}{dy^2} - (\sigma^2 - k^2) \bar{u} = 0, \quad \frac{d^2 \bar{\varphi}}{dy^2} - \sigma^2 \bar{\varphi} + k^2 \frac{\varepsilon_{15}}{\varepsilon_{11}} \bar{u} = 0, \quad y > 0, \quad (11)$$

$$\frac{d^2 \overline{\Phi^{(1)}}}{dy^2} - \sigma^2 \overline{\Phi^{(1)}} = 0, \quad y < 0, \quad (12)$$

where  $\bar{u}(\sigma, y) = \int_{-\infty}^{\infty} u(x, y) e^{i\sigma x} dx$ ,  $\bar{\varphi}(\sigma, y) = \int_{-\infty}^{\infty} \varphi(x, y) e^{i\sigma x} dx$ , and the boundary conditions will be

$$c_{44} \frac{d\bar{u}}{dy} + \varepsilon_{15} \frac{d\bar{\varphi}}{dy} - 2\pi i k c_{44} (1 + \chi) \sin \theta_0 \delta(\sigma - k \cos \theta_0) = 0, \quad y = 0, \quad (13)$$

$$y = 0, \quad \bar{\varphi} + 2\pi \frac{\varepsilon_{15}}{\varepsilon_{11}} \delta(\sigma - k \cos \theta_0) = \overline{\Phi^{(1)}} = \overline{\Phi^+}(\sigma), \quad (14)$$

$$y = 0, \quad \varepsilon_{15} d\bar{u} / dy - \varepsilon_{11} d\bar{\varphi} / dy + \varepsilon_0 d\overline{\Phi^{(1)}} / dy = \overline{\Psi^-}(\sigma), \quad (15)$$

$$y = -h, \quad \overline{\Phi^{(1)}} = 0. \quad (16)$$

Here  $\varepsilon_0$  is an electric constant of semi-space  $y < 0$  and  $\delta(x)$  is the Dirac delta-function. We also considered that  $\int_{-\infty}^{\infty} e^{i\alpha x} dx = 2\pi \delta(\alpha)$ .

To satisfy the condition of the outgoing wave, we assume that  $\gamma(\sigma) = \sqrt{\sigma^2 - k^2} \rightarrow |\sigma|$  when  $|\sigma| \rightarrow \infty$  and  $\sqrt{\sigma^2 - k^2} = -i\sqrt{k^2 - \sigma^2}$ , i.e. in the complex plane  $\alpha = \sigma + i\tau$  the real axis bypasses the branching points  $-k$  from top and  $k$  from bottom [3].

Whereas  $u, \varphi \rightarrow 0$  when  $y \rightarrow \infty$ , the solutions of equations (11) and (12), will have the form

$$\begin{aligned} \bar{u} &= A(\sigma) e^{-\gamma y}, \\ \bar{\varphi} &= B(\sigma) e^{-|\sigma|y} + \frac{\varepsilon_{15}}{\varepsilon_{11}} A(\sigma) e^{-\gamma y}, \quad y > 0, \end{aligned} \quad (17)$$

$$\overline{\Phi^{(1)}} = C(\sigma) \operatorname{ch} |\sigma| y + D(\sigma) \operatorname{sh} |\sigma| y, \quad y < 0.$$

From conditions (14)–(16) we obtain:

$$C(\sigma) = \overline{\Phi^+}(\sigma), \quad D(\sigma) = \overline{\Phi^+}(\sigma) \frac{\operatorname{ch} |\sigma| h}{\operatorname{sh} |\sigma| h},$$

$$B(\sigma) = -\frac{\varepsilon_0}{\varepsilon_{11}} \overline{\Phi^+}(\sigma) \frac{\operatorname{ch} |\sigma| h}{\operatorname{sh} |\sigma| h} + \frac{1}{\varepsilon_{11} |\sigma|} \overline{\Psi^-}(\sigma),$$

$$A(\sigma) = \left( \varepsilon_{11} + \varepsilon_0 \frac{\operatorname{ch} |\sigma| h}{\operatorname{sh} |\sigma| h} \right) \frac{\overline{\Phi^+}}{\varepsilon_{15}} - \frac{1}{\varepsilon_{15} |\sigma|} \overline{\Psi^-} - 2\pi \delta(\sigma - k \cos \theta_0).$$

From condition (13) we derive the following functional equation for unknown functions  $\overline{\Psi^-}(\sigma)$  and  $\overline{\Phi^+}(\sigma)$ :

$$\varepsilon_1 \overline{\Phi^+}(\sigma) - (1/|\sigma|)K(\sigma)\overline{\Psi^-}(\sigma) + 2\pi\lambda\delta(\sigma - k\cos\theta_0) = 0, \quad (18)$$

where  $\varepsilon_1 = (1 + \chi)\varepsilon_{11} + \varepsilon_0$ ,  $\lambda = 2 \frac{(1 + \chi)\varepsilon_{15} i \operatorname{tg} \theta_0}{K_2(k\cos\theta_0)}$ ,

$$K(\sigma) = K_1(\sigma)/K_2(\sigma), \quad K_1(\sigma) = (1 + \chi)\sqrt{\sigma^2 - k^2}/|\sigma| - \chi,$$

$$K_2(\sigma) = \frac{1}{\varepsilon_1} \left( (1 + \alpha) \frac{\sqrt{\sigma^2 - k^2}}{|\sigma|} \left( \varepsilon_{11} + \varepsilon_0 \frac{\operatorname{ch}|\sigma|h}{\operatorname{sh}|\sigma|h} \right) - \varepsilon_0 \chi \frac{\operatorname{ch}|\sigma|h}{\operatorname{sh}|\sigma|h} \right)$$

and  $K_{1,2}(\sigma) \rightarrow 1$  when  $|\sigma| \rightarrow \infty$ .

It is shown that  $K_1(\sigma)$  function has a zero when  $\sigma = \pm\sigma_1$ , where  $\sigma_1$  is the only root of equation  $K_1(\sigma) = 0$  when  $\sigma > k$ , and  $K_2(\sigma)$  function has a zero when  $\sigma = \pm\sigma_2$ , where  $\sigma_2$  is the only root of equation  $K_2(\sigma) = 0$  when  $k < \sigma_2 < \sigma_1$ , so  $K_i(k) < 0$ ,  $K_i(\infty) > 0$ ,  $K_i'(\sigma) > 0$  when  $\sigma > k$  ( $i = 1, 2$ ),  $K_2(\sigma) > 0$ .

The functional equation can be considered as the Riemann boundary problem in the theory of analytical functions.

The real axis bypasses also points  $\sigma = \sigma_1$ ,  $\sigma = \sigma_2$  from the top and  $\sigma = -\sigma_1$ ,  $\sigma = -\sigma_2$  from the bottom, ensuring the observance of the conditions of outgoing wave. So, as the dispersion equations  $K_i(\sigma) = 0$  ( $i = 1, 2$ ) have roots  $\pm\sigma_i$ , the localized (surface) waves propagate in the piezoelectric medium.

Let us find the solution of functional equation (18) by means of factorization method, representing  $K(\sigma)$  and  $|\sigma|$  in the following form [2]:

$$K(\sigma) = K^-(\sigma) K^+(\sigma), \quad |\sigma| = (\sigma - i0)^{1/2} (\sigma + i0)^{1/2}, \quad (19)$$

where  $K^\pm(\alpha)$  are regular functions that have no zeros in the upper and lower semi-spaces  $\operatorname{Im} \alpha > 0$  and  $\operatorname{Im} \alpha < 0$  of the complex plane, where  $\alpha = \sigma + i\tau$ . Note, that  $K^\pm(\alpha) \rightarrow 1$  are in their regularity domains when  $|\alpha| \rightarrow \infty$ . Then equation (18) is presented in the following form:

$$\frac{\varepsilon_1(\sigma + i0)^{1/2}}{K^+(\sigma)} \overline{\Phi^+}(\sigma) - \frac{1}{(\sigma - i0)^{1/2}} K^-(\sigma) \overline{\Psi^-}(\sigma) + 2\pi\lambda \frac{(k\cos\theta_0 + i0)^{1/2}}{K^+(k\cos\theta_0)} \delta(\sigma - k\cos\theta_0) = 0.$$

Taking into consideration also that

$$2\pi i \delta(\sigma - k\cos\theta_0) = \frac{1}{\sigma - k\cos\theta_0 - i0} - \frac{1}{\sigma - k\cos\theta_0 + i0},$$

equation (18) will be written in the following form when  $-\infty < \sigma < \infty$ :

$$J^+(\sigma) = \frac{\varepsilon_1(\sigma + i0)^{1/2}}{K^+(\sigma)} \overline{\Phi^+}(\sigma) - \frac{\lambda(k\cos\theta_0 + i0)^{1/2}}{iK^+(k\cos\theta_0)} \cdot \frac{1}{\sigma - k\cos\theta_0 + i0} =$$

$$= \frac{1}{(\sigma - i0)^{1/2}} K^-(\sigma) \overline{\Psi^-}(\sigma) - \frac{\lambda(k\cos\theta_0 + i0)^{1/2}}{iK^+(k\cos\theta_0)} \cdot \frac{1}{\sigma - k\cos\theta_0 - i0} = J^-(\sigma).$$

As this equation is solved by means of the same method as that in [2, 4], therefore, the equality  $J^+(\sigma) = J^-(\sigma)$  holds, only if  $J^+(\sigma) = J^-(\sigma) = 0$ .

Thus, we obtain for the unknown functions:

$$\begin{aligned}\overline{\Phi}^+(\sigma) &= \frac{\lambda(k\cos\theta_0 + i0)^{1/2}}{i\varepsilon_1 K^+(k\cos\theta_0)} \cdot \frac{1}{\sigma - k\cos\theta_0 + i0} \cdot \frac{K^+(\sigma)}{(\sigma + i0)^{1/2}}, \\ \overline{\Psi}^-(\sigma) &= \frac{\lambda(k\cos\theta_0 + i0)^{1/2}}{iK^+(k\cos\theta_0)} \cdot \frac{1}{\sigma - k\cos\theta_0 - i0} \cdot \frac{(\sigma - i0)^{1/2}}{K^-(\sigma)}.\end{aligned}$$

Hence, the problem of binding the expressions for amplitudes of displacement and electric field will have solutions in the following form:

$$\begin{aligned}w(x, y) &= w_\infty(x, y) + \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{u}(\sigma, y) e^{-i\sigma x} d\sigma, \\ \Phi(x, y) &= \Phi_\infty(x, y) + \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{\varphi}(\sigma, y) e^{-i\sigma x} d\sigma, \\ \Phi^{(1)}(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{\Phi}^{(1)}(\sigma, y) e^{-i\sigma x} d\sigma.\end{aligned}\tag{20}$$

The wave field in the piezoelectric medium is presented by the diffracted, incident, reflected and localized (surface) waves.

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**Գ. Ա. Շահբազյան, Հ. Ա. Ղազարյան.** Սահքի հարթ էլեկտրաառաձգական ալիքի դիֆրակցիան կիսաանվերջ շերտի վրա պիեզոէլեկտրական կիսատարածությունում էջ. 33–37

Դիտարկված է պիեզոէլեկտրիկ-վակուումային միջավայրում սահքի հարթ ալիքի դիֆրակցիան՝ բարակ կիսաանվերջ մետաղական շերտի վրա, հողակցված, մետաղական բարակ անվերջ շերտի (էլեկտրոդ) առկայության դեպքում: Խնդիրը բերված է անալիտիկ ֆունկցիաների տեսության ֆունկցիոնալ