INTERASSOCIATIVITY VIA HYPERIDENTITIES

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We study interassociativity of semigroups through the following hyperidentities of associativity ([1]-[3]):

$$X(Y(x,y),z) = Y(x,X(y,z)), \qquad (ass)_1$$

$$X(Y(x,y),z) = X(x,Y(y,z)), \qquad (ass)_2$$

$$X(X(x,y),z) = Y(x,Y(y,z)). \qquad (ass)_3$$

Moreover, in the q-algebras or e-algebras from $(ass)_3$ it follows $(ass)_2$ and from $(ass)_2$ it follows $(ass)_1$.

Definition. The semigroup $(S;\circ)$ is called $\{i,j\}$-interassociative to the semigroup $(S;\cdot)$ if algebra $S(\circ,\cdot)$ satisfies the hyperidentities $(ass)_i$ and $(ass)_j$, where $i,j = 1,2,3$. If $i=j$ the semigroup $(S;\circ)$ is called $\{i\}$-interassociative to $(S;\cdot)$.

We denote by $\mathrm{Int}_{\{i,j\}}(S;\cdot)$ the set of semigroups which are $\{i,j\}$-interassociative to semigroup $(S;\cdot)$. If $i=j$ the set $\mathrm{Int}_{\{i,j\}}(S;\cdot)$ is denoted by $\mathrm{Int}_{\{i\}}(S;\cdot)$.

Let $(\mathcal{F}(X);\cdot)$ be free semigroup generated by the set X, and $(\mathcal{F}C(X);\cdot)$ be the free commutative semigroup generated by the set X.

Theorem 1. $\mathrm{Int}_{\{1,2\}}(\mathcal{F}(X);\cdot) = \mathrm{Int}_{\{2\}}(\mathcal{F}(X);\cdot) = \{(\mathcal{F}(X);\cdot)\}$, *where* $|X| \geqslant 3$.

Theorem 2. $\mathrm{Int}_{\{3\}}(\mathcal{F}(X);\cdot) = \{(\mathcal{F}(X);\cdot)\}$.

Theorem 3. $\mathrm{Int}_{\{3\}}(\mathcal{F}C(X);\cdot) = \{(\mathcal{F}C(X);\cdot)\}$.

Theorem 4. $\mathrm{Int}_{\{2\}}(\mathcal{F}C(X);\cdot) = \{(\mathcal{F}C(X);*_x) \,|\, x \in \mathcal{F}C(X)\} \cup (\mathcal{F}C(X);\cdot)$, *where* $|X| \geqslant 4$, $a *_x b = axb$, $a,b \in \mathcal{F}C(X)$.

Theorem 5. *If* $|X| = 1$ *and* $X = \{a\}$, *then* $\mathrm{Int}_{\{1\}}(\mathcal{F}(X);\cdot) = \mathrm{Int}_{\{2\}}(\mathcal{F}(X);\cdot) = \{(\mathcal{F}(X);\cdot)\} \cup \{(\mathcal{F}(X);*_x) \,|\, x \in \mathcal{F}(X)\} \cup \{(\mathcal{F}(X);\circ)\}$, *where* $a^m \circ a^n = a^{m+n-1}$, $m,n \in \mathbb{N}$.

Using the result of [5] we prove the Theorem 1 for $|X| = 2$ streightforwardly.

In [4] is characterized $\mathrm{Int}_{\{1\}}(\mathcal{F}C(X);\cdot)$ and $\mathrm{Int}_{\{1,2\}}(\mathcal{F}C(X);\cdot)$. In [5] is considered $\mathrm{Int}_{\{1\}}(\mathcal{F}(X);\cdot)$.

References

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