LOADS TRANSFER FROM FINITE NUMBER FINITE STRINGERS TO AN INFINITE SHEET THROUGH ADHESIVE SHEAR LAYERS

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Abstract. In this article the problem for an elastic infinite sheet which is strengthened by finite number finite stringers with different modulus of elasticity and small constant thickness is considered. The contact interaction between sheet and stringers through adhesive layers having different physical-mechanical properties and geometric configurations was realized.

Let an infinite sheet which defined in generalized plane stress state with small constant thickness d, the Young's modulus E and the Poisson's ratio ν, on its surface along at $y=0$ (xOy is its average plane) line on the $\left[a_j,b_j\right]\left(b_j>a_j,\ j=\overline{1,n};\ \ b_j<a_{j+1},\ j=\overline{1,n-1}\right)$ finite intervals is strengthened by arbitrary finite number of finite stringers. It is supposed that the stringers having rectangular cross-sections with areas $F_j,\ j=\overline{1,n}$ with small different thickness h_j $\left(h_j<<b_j-a_j;\ j=\overline{1,n}\right)$, and constant width $b^*\left(b^*<<b_j-a_j\ \left(j=\overline{1,n}\right)\right)$ with different modulus of elasticity E_j $\left(j=\overline{1,n}\right)$, where $x\in\left[a_j,b_j\right],\ j=\overline{1,n}$. The interaction between infinite sheet and stringers was realized through thin, uniform, elastic adhesive layers with Young's modulus E_k, the Poisson's ratio ν_k and small constant thickness h_k. The problem is to specify the law of distribution of unknown shear stresses which are acting between sheet and stringers when concentrated forces P_j $\left(j=\overline{1,n}\right)$ are applied at one end points of stringers $x=b_j,\ j=\overline{1,n}$, and directed along the Ox axis.

It is supposed that for the stringers the model of uniaxial strain state in combination with the model of contact along the line is realized, and for the adhesive layers it is the pure shear conditions (see [1]), that is the interaction between infinite sheet and stringers as a line loading of the sheet is idealized.

Taking into account above assumptions in [1], the determination problem of unknown shear stresses is reduced to the solution of the following system of Fredholm's integral equations of the second kind with respect to unknown functions $\psi_j\left(x\right)$ $\left(j=\overline{1,n}\right)$, which specified in different finite intervals:

$$\psi_j\left(x\right)+\tilde{\delta}^2\sum_{i=1}^{n}\int_{\alpha_i}^{\beta_i}K_j\left(x,t\right)\psi_i\left(t\right)dt=f_0^{(j)}\left(x\right),\quad \alpha_j\le x\le\beta_j,\ \ j=\overline{1,n}, \tag{1.1}$$

where

$$\psi_j\left(x\right)=ap_j\left(ax\right)\ \left(j=\overline{1,n}\right),\ p_j\left(x\right)=b^*\tau_j\left(x\right),\ \tilde{\gamma}_j^2=b^*G_k/h_kE_jF_j,\ \ j=\overline{1,n},$$

$$\tilde{\delta}^2=ab^*\left(1+\nu\right)\left(3-\nu\right)G_k/4\pi Edh_k,\ G_k=E_k/2\left(1+\nu_k\right),\ \alpha_j=a_j/a,\ \beta_j=b_j/a,$$

$$K_j\left(x,t\right)=\ln\frac{1}{\left|x-t\right|}-a\tilde{\gamma}_j^2\int_{\alpha_j}^{\beta_j}G_j\left(ax,as\right)\ln\frac{1}{\left|s-t\right|}ds,\quad j=\overline{1,n},$$

$$f_0^{(j)}\left(x\right)=\frac{P_jb^*a\tilde{\gamma}_j\operatorname{ch}\left[a\tilde{\gamma}_j\left(x-\alpha_j\right)\right]}{\operatorname{sh}\left[a\tilde{\gamma}_j\left(\beta_j-\alpha_j\right)\right]},\quad j=\overline{1,n},$$

$$G_j\left(x,s\right)=\frac{1}{\tilde{\gamma}_j\operatorname{sh}\left[\tilde{\gamma}_j\left(b_j-a_j\right)\right]}\begin{cases}\operatorname{ch}\tilde{\gamma}_j\left(x-b_j\right)\operatorname{ch}\tilde{\gamma}_j\left(s-a_j\right), & x>s,\\ \operatorname{ch}\tilde{\gamma}_j\left(x-a_j\right)\operatorname{ch}\tilde{\gamma}_j\left(s-b_j\right), & x<s,\quad j=\overline{1,n},\end{cases}$$

$G_j\left(x,s\right)=G_j\left(s,x\right),\ j=\overline{1,n}$, are Green's functions which satisfying the following equalities:

$$\int_{a_j}^{b_j} G_j(x,s)\cos\left[\frac{m\pi(s-a_j)}{b_j-a_j}\right]ds = \frac{(b_j-a_j)^2}{(b_j-a_j)^2\tilde{\gamma}_j^2+m^2\pi^2}\cos\left[\frac{m\pi(x-a_j)}{b_j-a_j}\right],\ m=0,1,2,...,\ j=\overline{1,n},$$

where the functions $\cos\left[\frac{m\pi(x-a_j)}{b_j-a_j}\right]$ $(m=0,1,2,...)$ $\left(j=\overline{1,n}\right)$ form full orthogonal systems in spaces $L_2(a_j,b_j)$, $j=\overline{1,n}$, the kernels $K_j(x,t)$, $j=\overline{1,n}$ of the system of integral equations (1.1) are square integrable functions by two variables and $\tau_j(x)$, $j=\overline{1,n}$, are unknown shear stresses which are acting between sheet and stringers.

It is shown that in the certain domain of the change of characteristic parameter of the problem this system of integral equations (1.1) in Banach space may be solved by the method of successive approximations. The particular cases are observed and the character and behaviour of unknown shear stresses are investigated. The numerical results for the solutions of (1.1) for two cases ($n=1$ and 2) are presented.

REFERENCE

1. Lubkin J.L. and Lewis L.C. Adhesive Shear Flow for an Axially Loaded, Finite Stringer Bonded to an Infinite Sheet. Quarterly Journal of Mechanics and Applied Mathematics, vol. 23, 1970.

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