

Scalar-Isovector δ Meson in the Relativistic Mean Field Theory and the Structure of Neutron Stars with a Quark Core¹

G. B. Alaverdyan*

Wave Processes Department, Yerevan State University, A. Manukian St. 1, 0025 Yerevan, Armenia

Received November 3, 2008

Abstract—In the framework of the relativistic mean field theory, we have considered the equation of state of superdense nuclear matter, taking into account an effective scalar-isovector δ meson field. The effect of the δ meson field on the characteristics of a Maxwell-type quark phase transition has been studied. The quark phase is described with the aid of the improved version of the MIT (Massachusetts Institute of Technology) bag model, in which interactions between the u , d , s quarks inside the bag are taken into account in the one-gluon exchange approximation. For different values of the bag parameter B , series of neutron star models with a quark core have been built. Stability problems for neutron stars with an infinitesimal quark core are discussed. An estimate is obtained for the amount of energy released in a catastrophic transformation of a critical neutron star to a star with a finite-size quark core.

PACS numbers: 97.60.Jd, 26.60.+c, 12.39.Ba

DOI: 10.1134/S0202289309010022

1. INTRODUCTION

The properties of such compact objects as neutron stars (NS) depend functionally on the equation of state (EOS) of matter in a sufficiently wide range of densities. A knowledge of the structure characteristics and the constituent composition of matter at extremely large densities is a necessary condition for revealing the physical nature of the NS internal structure and integral parameters. One of the effectively used theories, sufficiently adequately describing the properties of nuclear matter as a system of strongly interacting baryons and mesons, is the relativistic mean field (RMF) theory [1, 2]. This theory has made it possible to obtain results which satisfactorily describe the structure of finite nuclei [3], the EOS of nuclear matter [4], and the features of heavy ion scattering [5]. Inclusion of a scalar-isovector δ meson into the scheme and a study of its role for asymmetric nuclear matter in the low density range has been conducted in [6, 7, 8]. An objective of this paper is to study the EOS of superdense nuclear matter in the framework of the RMF theory and an investigation of changes in the parameters of the first-order phase transition due to inclusion of δ meson exchange. Using the EOS obtained, we calculate the integral and structure characteristics of NS with a quark core.

2. PROPERTIES OF NEUTRON STAR MATTER

2.1. Nuclear Matter

The nonlinear Lagrangian density of an interacting multiparticle system consisting of nucleons and an isoscalar-scalar σ meson, an isovector-scalar δ meson and an isovector-vector ρ meson, an isoscalar-vector ω meson as well as free electrons has the following form in quantum hadrodynamics (QHD)¹:

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_N \left[\gamma^\mu \left(i\partial_\mu - g_\omega \omega_\mu(x) - \frac{1}{2} g_\rho \vec{\tau} \cdot \vec{\rho}_\mu(x) \right) \right. \\ & \left. - \left(m_N - g_\sigma \sigma(x) - g_\delta \vec{\tau} \cdot \vec{\delta}(x) \right) \right] \psi_N \\ & + \frac{1}{2} (\partial_\mu \sigma(x) \partial^\mu \sigma(x) - m_\sigma \sigma(x)^2) \\ & - \frac{b}{3} m_N (g_\sigma \sigma(x))^3 - \frac{c}{4} (g_\sigma \sigma(x))^4 \\ & + \frac{1}{2} m_\omega^2 \omega^\mu(x) \omega_\mu(x) + \frac{1}{2} m_\rho^2 \vec{\rho}^\mu(x) \vec{\rho}_\mu(x) \\ & - \frac{1}{4} \Omega_{\mu\nu}(x) \Omega^{\mu\nu}(x) - \frac{1}{4} R_{\mu\nu}(x) R^{\mu\nu}(x) \\ & + \frac{1}{2} (\partial_\mu \vec{\delta}(x) \partial^\mu \vec{\delta}(x) - m_\delta^2 \vec{\delta}(x)^2) \\ & + \bar{\psi}_e (i\gamma^\mu \partial_\mu - m_e) \psi_e. \end{aligned} \quad (1)$$

*E-mail: galaverdyan@ysu.am

¹Talk given at the International Conference RUSGRAV-13, June 23–28, 2008, PFUR, Moscow.

¹We will use the natural system of units $\hbar = c = 1$.

Table 1. Constants of the theory without $(\sigma\omega\rho)$ and with $(\sigma\omega\rho\delta)$ the δ meson field

	a_σ, fm^2	a_ω, fm^2	a_δ, fm^2	a_ρ, fm^2	b, fm^{-1}	c
$\sigma\omega\rho$	9.154	4.828	0	4.794	1.654×10^{-2}	1.319×10^{-2}
$\sigma\omega\rho\delta$	9.154	4.828	2.5	13.621	1.654×10^{-2}	1.319×10^{-2}

Here, $g_\sigma, g_\omega, g_\delta$ and g_ρ denote the coupling constants of the nucleon with the corresponding meson. In the RMF theory, the meson fields $\sigma(x), \omega_\mu(x), \vec{\delta}(x)$, and $\vec{\rho}_\mu(x)$ are replaced by the (effective) fields $\bar{\sigma}, \bar{\omega}_\mu, \bar{\vec{\delta}}$, and $\bar{\vec{\rho}}_\mu$. Re-denoting the meson fields and coupling constants according to

$$\begin{aligned} g_\sigma \bar{\sigma} &\equiv \sigma, & g_\omega \bar{\omega}_0 &\equiv \omega, \\ g_\delta \bar{\vec{\delta}}^{(3)} &\equiv \delta, & g_\rho \bar{\vec{\rho}}_0^{(3)} &\equiv \rho, \end{aligned} \quad (2)$$

$$\begin{aligned} (g_\sigma/m_\sigma)^2 &\equiv a_\sigma, & (g_\omega/m_\omega)^2 &\equiv a_\omega, \\ (g_\delta/m_\delta)^2 &\equiv a_\delta, & (g_\rho/m_\rho)^2 &\equiv a_\rho \end{aligned} \quad (3)$$

and introducing the asymmetry parameter

$$\alpha = (n_n - n_p)/n, \quad (4)$$

one can present the field equations in the form

$$\begin{aligned} \sigma &= a_\sigma (n_{sp}(n, \alpha) + n_{sn}(n, \alpha) \\ &\quad - bm_N \sigma^2 - c\sigma^3), \end{aligned} \quad (5)$$

$$\omega = a_\omega n, \quad (6)$$

$$\delta = a_\delta (n_{sp}(n, \alpha) - n_{sn}(n, \alpha)), \quad (7)$$

$$\rho = -\frac{1}{2} a_\rho n \alpha, \quad (8)$$

where

$$n_{sp}(n, \alpha) = \frac{1}{\pi^2} \quad (9)$$

$$\times \int_0^{k_F(n)(1-\alpha)^{1/3}} \frac{m_p^*(\sigma, \delta)}{\sqrt{k^2 + m_p^*(\sigma, \delta)^2}} k^2 dk,$$

$$n_{sn}(n, \alpha) = \frac{1}{\pi^2} \quad (10)$$

$$\times \int_0^{k_F(n)(1+\alpha)^{1/3}} \frac{m_n^*(\sigma, \delta)}{\sqrt{k^2 + m_n^*(\sigma, \delta)^2}} k^2 dk,$$

$$k_F(n) = \left(\frac{3\pi^2 n}{2} \right)^{1/3}. \quad (11)$$

The effective masses of the proton and neutron are determined by the expressions

$$\begin{aligned} m_p^*(\sigma, \delta) &= m_N - \sigma - \delta, \\ m_n^*(\sigma, \delta) &= m_N - \sigma + \delta. \end{aligned} \quad (12)$$

If the constants a_ω and a_ρ are known, equations (6) and (8) determine the functions $\omega(n)$ and $\rho(n, \alpha)$. Moreover, a knowledge of the other constants a_σ, a_δ, b , and c makes it possible to solve the set of equations (5), (7), (9), (10) in a self-consistent way and to determine the remaining two meson field functions $\sigma(n, \alpha)$ and $\delta(n, \alpha)$. The standard QHD procedure makes it possible to obtain expressions for the energy density $\varepsilon(n, \alpha)$ and pressure $P(n, \alpha)$ [9].

The constants of the theory are numerically defined in such a way as to reproduce the empirically known values of such nuclear characteristics at saturation as the bare nucleon mass $m_N = 938.93$ MeV, the parameter $\gamma = m_N^*/m_N = 0.78$, the saturated nuclear matter baryon number density $n_0 = 0.153$ fm⁻³, the specific binding energy $f_0 = -16.3$ MeV, the compressibility modulus $K = 300$ MeV, and the asymmetry energy $E_{\text{sym}}^{(0)} = 32.5$ MeV. To reveal the role of taking into account the δ meson, we have used the value $a_\delta = 2.5$ fm² (see [10]). Table 1 shows the values of constants of the theory obtained without taking into account the interaction channel due to the isovector-scalar δ meson ($\sigma\omega\rho$) and taking it into account ($\sigma\omega\rho\delta$). Figure 1 represents a three-dimensional picture of the energy per baryon $E_b(n, \alpha) = \varepsilon_{NM}/n$ vs. the baryon number density n and the asymmetry parameter α in the case of a β -equilibrium charged “npe-” plasma. The lines correspond to different fixed values of charge per baryon. The thick line corresponds to a β -equilibrium, electrically neutral “npe-” matter. The lower surface corresponds to the “ $\sigma\omega\rho$ ” model, the upper one to the “ $\sigma\omega\rho\delta$ ” model. It is seen that inclusion of the δ meson field increases the energy per nucleon value, and this change is enhanced as the nuclear matter asymmetry parameter increases. The results of our analysis show that the scalar-isovector δ -field leads to an increased stiffness of the nuclear matter EOS due to splitting the effective masses of the proton and the neutron thus increasing the asymmetry energy. The EOS obtained by us in the

Table 2. Parameters of the Maxwell phase transition for different values of the “bag” constant B without taking into account the δ meson field ($\sigma\omega\rho$) and with it ($\sigma\omega\rho\delta$)

B		μ_b	n_N	n_Q	P_0	ε_N	ε_Q	λ
MeV/fm ³		MeV	fm ⁻³	fm ⁻³	MeV/fm ³	MeV/fm ³	MeV/fm ³	
60	$\sigma\omega\rho$	965.4	0.1220	0.2826	1.965	115.8	270.9	2.299
60	$\sigma\omega\rho\delta$	965.9	0.1207	0.2831	2.11	114.5	271.4	2.327
69.3	$\sigma\omega\rho$	1037	0.246	0.3557	15.57	239.6	353.4	1.385
69.3	$\sigma\omega\rho\delta$	1032	0.2241	0.3504	13.84	217.5	347.9	1.504
80	$\sigma\omega\rho$	1142	0.3792	0.4819	48.54	384.5	501.8	1.159
80	$\sigma\omega\rho\delta$	1119	0.3276	0.4525	37.95	328.8	468.6	1.278
100	$\sigma\omega\rho$	1298	0.5506	0.7175	121.3	593.4	810	1.133
100	$\sigma\omega\rho\delta$	1257	0.4746	0.6497	93.30	503.3	723.5	1.213
120	$\sigma\omega\rho$	1396	0.6512	0.8975	180.2	728.9	1073	1.18
120	$\sigma\omega\rho\delta$	1354	0.5729	0.8165	143.9	631.7	961.4	1.24

normal nuclear density range is matched to the well-known Baym-Bethe-Pethick (BBP) EOS [11].

2.2. Strange Quark Matter

To describe the quark phase, we have used the improved version of the MIT (Massachusetts Institute of Technology) bag model [12], in which interactions between the u , d , s quarks inside the bag are taken into account in the one-gluon exchange approximation [13]. The quark phase consists of the three quark flavors u , d , s and electrons in equilibrium with respect to weak interactions. For the quark masses we have used the values $m_u = 5$ MeV, $m_d = 7$ MeV and $m_s = 150$ MeV, while for the strong interaction constant we have taken the value $\alpha_s = 0.5$. The calculations were conducted for different values of the bag parameter B in the range of 60 to 120 MeV/fm³.

2.3. Phase Transition to Quark Matter under Constant Pressure

We assume in this paper that the nuclear to quark matter conversion is the ordinary first-order phase transition described by the Maxwell construction. A separate paper will be devoted to a study of alterations in the characteristics of a phase transition with mixed phase formation [14] due to inclusion of the δ -meson field as well as the effect of these alterations on the integral and structure parameters of hybrid stars.

In the case of an ordinary first-order phase transition, it is supposed that both the nuclear and quark

matter, taken separately, is electrically neutral, and at a certain value of the pressure, P_0 , corresponding to co-existence of the two phases, the baryonic chemical potentials of the two phases coincide. Table 2 repre-

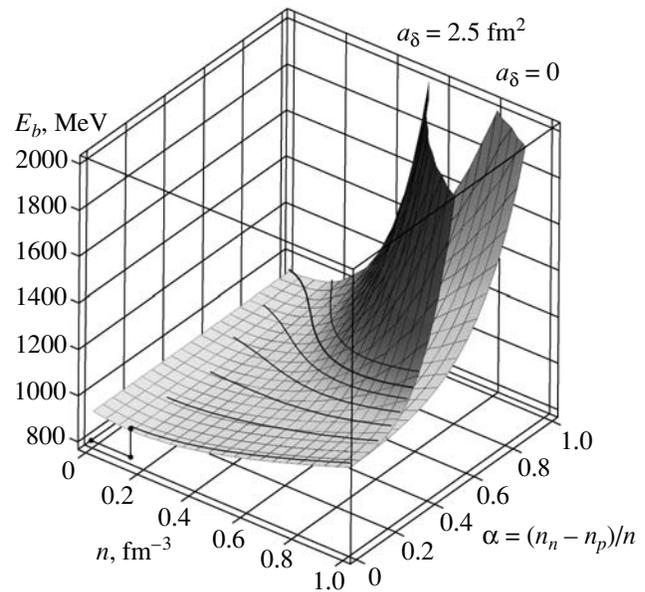


Fig. 1. A three-dimensional representation of the energy per baryon E_b as a function of the baryon number density n and the asymmetry parameter α in the case of a β -equilibrium charged “npe-” plasma. The upper surface corresponds to the “ $\sigma\omega\rho\delta$ ” model, the lower one to the “ $\sigma\omega\rho$ ” model.

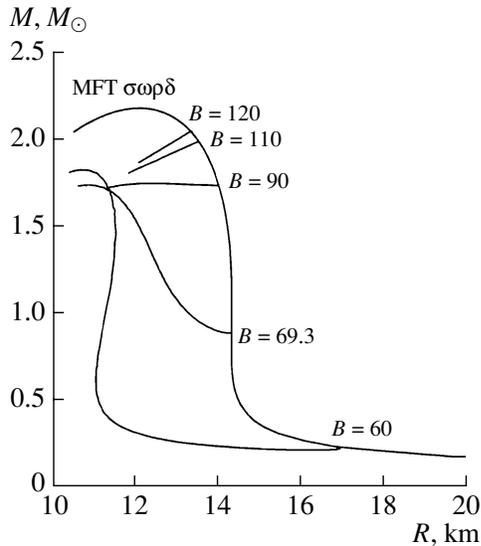


Fig. 2. The stellar mass M vs. radius R for different values of the bag parameter B in the “ $\sigma\omega\rho\delta$ ” model. The values of B in MeV/fm^3 are shown near the critical configurations corresponding to quark phase formation.

sents the parameter values of such a phase transition for five different values of the “bag” parameter B both with and without the δ field. In this table, μ_b is the baryonic chemical potential at the phase transition point, n_N and n_Q are the baryon number densities of the nuclear and quark matter, respectively, at the transition point, ε_N and ε_Q are the energy densities, P_0 is the phase transition pressure, and $\lambda = \varepsilon_Q/(\varepsilon_N + P_0)$ is the density jump parameter. It can be seen that inclusion of the δ interaction channel results in decreased values of the transition pressure P_0 and baryon number densities of both phases at the phase transition point, n_N and n_Q . Meanwhile, the value of the jump parameter λ increases.

3. MODELS OF NEUTRON STARS WITH A QUARK CORE

Using the EOS of nuclear matter obtained in the previous section we have integrated the Tolman-Oppenheimer-Volkoff (TOV) set of equations [15] and obtained both the structure functions $\varepsilon(r)$, $P(r)$ and $m(r)$ and the stellar integral parameters: the gravitational mass M and the radius R for different values of the central pressure P_c .

Figure 2 shows the $M(R)$ dependence for different values of the bag parameter B . It is seen that the purely nucleon EOS leads to a maximum mass of $\sim 2.2 M_\odot$. The advent of a quark phase in the NS diminishes this value up to $\sim 1.75 M_\odot$. According to [16], in the case $\lambda > \lambda_{\text{cr}} = 3/2$, an infinitesimal

core of the new phase is unstable. Our analysis shows that, for $B < 69.3 \text{ MeV}/\text{fm}^3$, the density jump parameter has the values $\lambda > \lambda_{\text{cr}}$, and NS configurations with infinitesimal quark cores will be unstable. In the latter case, there is a nonzero minimum value of the quark core radius of a neutron star. Accretion of matter on a critical neutron star will then result in a catastrophic (jumplike) transformation of the star, forming a star with a quark core of finite size. The catastrophic transformation with finite quark core formation at the stellar center will release an enormous amount of energy, compared with energy release at a Supernova explosion. Our calculations have allowed an evaluation of this energy. Thus, for $B = 60 \text{ MeV}/\text{fm}^3$, a star of mass $M \approx 0.24 M_\odot$ and radius $R \approx 16.77 \text{ km}$ forms, in a jumplike manner, a star of radius $R \approx 13.95 \text{ km}$ and a quark core, having a core mass $M_{\text{core}} \approx 0.087 M_\odot$ and radius $R_{\text{core}} \approx 4.38 \text{ km}$. The catastrophic restructuring process releases an energy $E_{\text{conv}} \approx 4 \times 10^{43} \text{ J}$. For $B > 90 \text{ MeV}/\text{fm}^3$, the quark degrees of freedom make the neutron star unstable, and configurations with a quark core are absent. In the case $69.3 \leq B \leq 90 \text{ MeV}/\text{fm}^3$, configurations with arbitrarily small quark cores are stable.

4. CONCLUSION

In the framework of the RMF theory, we have considered the EOS of NS matter and obtained that inclusion of the isovector-scalar δ meson field leads to nonnegligible changes in the characteristics of quark first-order phase transitions. For different values of the bag parameter B , we have constructed neutron star models with quark cores. The results have shown that, depending on value of this parameter realized in the Nature, the following criteria may take place. In the case $B > 90 \text{ MeV}/\text{fm}^3$, the quark phase leads to a total neutron star instability, and stars with quark cores are absent. In the case $69.3 \leq B \leq 90 \text{ MeV}/\text{fm}^3$, stable NS with arbitrarily small quark cores may exist. If $B < 69.3 \text{ MeV}/\text{fm}^3$, then configurations with small quark cores are unstable. In the latter case, accretion of matter will result in a catastrophic NS transformation after which a quark core of finite size will be formed.

ACKNOWLEDGMENTS

This work has been supported by the Ministry of Education and Science of the Republic of Armenia, Grant 2008-130.

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