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P h y s i c s

ON FUSION MATRIX IN $N = 1$ SUPER LIUVILLE FIELD THEORY

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We study several aspects of the $N = 1$ super Liouville theory. We show that certain elements of the fusion matrix in the Neveu–Schwarz sector are related to the structure constants according to the same rules, which we observe in rational conformal field theory.

***Keywords*:** conformal field theory, Moore–Seiberg relations.

Introduction. In this paper we study some of the Moore–Seiberg relations [1] for the fusion matrix of the $N = 1$ super Liouville field theory. Recall some basic facts on the fusion matrix. It is defined as a matrix of transformation of conformal blocks [2] in s and t channels:

$$\mathcal{F}^s_p \begin{bmatrix} k & j \\ i & l \end{bmatrix} = \sum_q F_{p,q} \begin{bmatrix} k & j \\ i & l \end{bmatrix} \mathcal{F}^t_q \begin{bmatrix} l & j \\ i & k \end{bmatrix}. \tag{1}$$

Here we write all formulas in the absence of the multiplicities, i.e. for the fusion numbers $N^i_{jk} = 0,1$. Our task here is to study the following relations [1], proved in rational CFT, in $N = 1$ super Liouville field theory:

$$F_{0,i} \begin{bmatrix} j & k \\ j & k^* \end{bmatrix} F_{i,0} \begin{bmatrix} k^* & k \\ j & j \end{bmatrix} = \frac{F_j F_k}{F_i}, \tag{2}$$

where

$$F_i \equiv F_{0,0} \begin{bmatrix} i & i^* \\ i & i \end{bmatrix} = \frac{S_{00}}{S_{0i}}, \tag{3}$$

and

$$C^p_{ij} = \frac{\eta_i \eta_j}{\eta_0 \eta_p} F_{0,p} \begin{bmatrix} j & i \\ j & i^* \end{bmatrix}, \quad \eta_i = \sqrt{C_{ii^*}/F_i}, \tag{4}$$

which using (2) can be written also as

$$C^p_{ij} = \frac{\xi_i \xi_j}{\xi_0 \xi_p} \frac{1}{F_{p,0} \begin{bmatrix} j^* & j \\ i & i \end{bmatrix}}, \quad \xi_i = \eta_i F_i = \sqrt{C_{ii^*} F_i}. \tag{5}$$

Let us explain notations. First of all 0 denotes vacuum field and i^* is the field conjugate to i in a sense that $N^0_{ii*} = 1$. Then S_{ij} is a matrix of the modular transformations, C^p_{ij} are structure constants, C_{ii^*} are two-point functions.

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The relation (2) is a consequence of the pentagon identity for fusion matrix [1]. The expression (3) results from the two different ways of calculation of the quantum dimension [1]. The Eqs. (4) and (5) result from the bootstrap equation combined with the pentagon identity [3].

These relations were examined in the Liouville field theory. The Eq. (2) in the Liouville field theory was tested in [4]. The expressions (4) and (5) were examined in the Liouville field theory in [3, 5]. In [3], (4) and (5) in the Liouville field theory were checked using the relation of the fusion matrix with boundary three-point function. In [5], Eq. (4) was checked using the following star-triangle integral identity for double Sine-functions $S_b(x)$:

$$\int \frac{dx}{i} \prod_{i=1}^{3} S_b(x+a_i) S_b(-x+b_i) = \prod_{i,j=1} S_b(a_i+b_j)\,, \tag{6}$$

where $\sum_i (a_i+b_i) = Q$. Recently it was found a super-symmetric generalization of this formula [6] (see Eq. (29)).

Our first aim here is to calculate the elements of the fusion matrix in the NS sector constructed in [7] with one of the intermediate entries set to the vacuum. Using the supersymmetric version of the star-triangle identity (29), we found that the elements of the fusion matrix with one of the entries set to the vacuum give rise to the corresponding structure constant according to the pattern of the Eqs. (4) and (5).

$N=1$ **Super Liouville Field Theory.** $N=1$ super Liouville field theory is defined on a two-dimensional surface with metric g_{ab} by the local Lagrangian density

$$\mathcal{L} = \frac{1}{2\pi} g_{ab}\partial_a\varphi\partial_b\varphi + \frac{1}{2\pi}(\psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi}) + 2i\mu b^2\bar{\psi}\psi e^{b\varphi} + 2\pi\mu^2 b^2 e^{2b\varphi}\,. \tag{7}$$

The energy-momentum tensor and the superconformal current are

$$T = -\frac{1}{2}(\partial\varphi\partial\varphi - Q\partial^2\varphi + \psi\partial\psi)\,, \quad G = i(\psi\partial\varphi - Q\partial\psi)\,. \tag{8}$$

The superconformal algebra is

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n}\,, \tag{9}$$

$$[L_m, G_k] = \frac{m-2k}{2}G_{m+k}\,, \tag{10}$$

$$\{G_k, G_l\} = 2L_{l+k} + \frac{c}{3}\left(k^2 - \frac{1}{4}\right)\delta_{k+l} \tag{11}$$

with the central charge

$$c_L = \frac{3}{2} + 3Q^2\,, \quad \text{where} \quad Q = b + \frac{1}{b}\,. \tag{12}$$

Here k and l take integer values for the Ramond sector and half-integer values for the Neveu–Schwarz sector. NS-NS primary fields $N_\alpha(z,\bar{z})$ in this theory, $N_\alpha(z,\bar{z}) = e^{\alpha\varphi(z,\bar{z})}$, have conformal dimensions $\Delta_\alpha^{NS} = 1/2\alpha(Q-\alpha)$.

Introduce also the field

$$\tilde{N}_\alpha(z,\bar{z}) = G_{-1/2}\bar{G}_{-1/2}N_\alpha(z,\bar{z})\,. \tag{13}$$

The R-R is defined as

$$R_\alpha(z,\bar{z}) = \sigma(z,\bar{z})e^{\alpha\varphi(z,\bar{z})}\,, \tag{14}$$

where σ is the spin field and has the dimension $\Delta_\alpha^R = 1/16 + \alpha(Q-\alpha)/2$.

The NS-NS and R-R operators with the same conformal dimensions are proportional to each other, namely we have

$$N_\alpha = \mathcal{G}_{\rm NS}(\alpha)N_{Q-\alpha}\,, \quad R_\alpha = \mathcal{G}_{\rm R}(\alpha)R_{Q-\alpha}\,, \tag{15}$$

where $\mathcal{G}_{\rm NS}(\alpha)$ and $\mathcal{G}_{\rm R}(\alpha)$ are so called reflection functions. They also give two-point functions. The elegant way to write the reflection functions is to introduce NS and R generalization of the ZZ function in the bosonic Liouville theory:

$$W_{\rm NS}(\alpha)=\frac{2(\pi\mu\gamma(bQ/2))^{-\frac{Q-2\alpha}{2b}}\pi(\alpha-Q/2)}{\Gamma(1+b(\alpha-Q/2))\Gamma(1+\frac{1}{b}(\alpha-Q/2))}, \tag{16}$$

$$W_{\rm R}(\alpha)=\frac{2\pi(\pi\mu\gamma(bQ/2))^{-\frac{Q-2\alpha}{2b}}}{\Gamma(1/2+b(\alpha-Q/2))\Gamma(1/2+\frac{1}{b}(\alpha-Q/2))}. \tag{17}$$

The reflection functions can be written:

$$\mathcal{G}_{\rm NS}(\alpha)=\frac{W_{\rm NS}(Q-\alpha)}{W_{\rm NS}(\alpha)},\quad \mathcal{G}_{\rm R}(\alpha)=\frac{W_{\rm R}(Q-\alpha)}{W_{\rm R}(\alpha)}. \tag{18}$$

The physical delta function normalizable states have $\alpha=Q/2+iP$.

The structure constants in the NS sector of $N=1$ super Liouville field theory are [8]:

$$C_{\rm NS}(\alpha_1,\alpha_2,\alpha_3)=\lambda^{(Q-\sum_{i=1}^3\alpha_i)/b}\times \frac{\Upsilon'_{\rm NS}(0)\Upsilon_{\rm NS}(2\alpha_1)\Upsilon_{\rm NS}(2\alpha_2)\Upsilon_{\rm NS}(2\alpha_3)}{\Upsilon_{\rm NS}(\alpha_1+\alpha_2+\alpha_3-Q)\Upsilon_{\rm NS}(\alpha_1+\alpha_2-\alpha_3)\Upsilon_{\rm NS}(\alpha_2+\alpha_3-\alpha_1)\Upsilon_{\rm NS}(\alpha_3+\alpha_1-\alpha_2)}, \tag{19}$$

$$\tilde{C}_{\rm NS}(\alpha_1,\alpha_2,\alpha_3)=\lambda^{(Q-\sum_{i=1}^3\alpha_i)/b}\times \frac{\Upsilon'_{\rm NS}(0)\Upsilon_{\rm NS}(2\alpha_1)\Upsilon_{\rm NS}(2\alpha_2)\Upsilon_{\rm NS}(2\alpha_3)}{\Upsilon_{\rm R}(\alpha_1+\alpha_2+\alpha_3-Q)\Upsilon_{\rm R}(\alpha_1+\alpha_2-\alpha_3)\Upsilon_{\rm R}(\alpha_2+\alpha_3-\alpha_1)\Upsilon_{\rm R}(\alpha_3+\alpha_1-\alpha_2)} \tag{20}$$

and $\lambda=\pi\mu\gamma(bQ/2)\,b^{1-b^2}$.

Fusion matrix in the NS sector is computed in [7]. Let us denote

$$\begin{bmatrix}\alpha_i & \alpha_j\\ \alpha_k & \alpha_l\end{bmatrix} \text{ as } \begin{bmatrix}\alpha_{ijkl}\end{bmatrix},\ \begin{bmatrix}N_{\alpha_i} & N_{\alpha_j}\\ N_{\alpha_k} & N_{\alpha_l}\end{bmatrix} \text{ as } \begin{bmatrix}N_{\alpha_{ijkl}}\end{bmatrix} \tag{21}$$

and

$$F_{\alpha_s,\alpha_t}\Big[\alpha_{3241}\Big]_1^1\equiv F_{N_{\alpha_s},N_{\alpha_t}}\Big[N_{\alpha_{3241}}\Big],\ F_{\alpha_s,\alpha_t}\Big[\alpha_{3241}\Big]_1^2\equiv F_{N_{\alpha_s},\tilde{N}_{\alpha_t}}\Big[N_{\alpha_{3241}}\Big], \tag{22}$$

$$F_{\alpha_s,\alpha_t}\Big[\alpha_{3241}\Big]_2^1\equiv F_{\tilde{N}_{\alpha_s},N_{\alpha_t}}\Big[N_{\alpha_{3241}}\Big],\ F_{\alpha_s,\alpha_t}\Big[\alpha_{3241}\Big]_2^2\equiv F_{\tilde{N}_{\alpha_s},\tilde{N}_{\alpha_t}}\Big[N_{\alpha_{3241}}\Big]. \tag{23}$$

To write the fusion matrix, we use the following convention. The functions Υ_i,Γ_i,S_i will be understood $\Upsilon_{\rm NS},\Gamma_{\rm NS},S_{\rm NS}$ for $i=1 \bmod 2$, and $\Upsilon_{\rm R},\Gamma_{\rm R},S_{\rm R}$ for $i=0 \bmod 2$. For review of these functions the last section of this paper. Now we can write the fusion matrix:

$$\begin{aligned}F_{\alpha_s,\alpha_t}\Big[\alpha_{3241}\Big]_j^i=&\frac{\Gamma_i(2Q-\alpha_t-\alpha_2-\alpha_3)\Gamma_i(Q-\alpha_t+\alpha_3-\alpha_2)\Gamma_i(Q+\alpha_t-\alpha_2-\alpha_3)}{\Gamma_j(2Q-\alpha_1-\alpha_s-\alpha_2)\Gamma_j(Q-\alpha_s-\alpha_2+\alpha_1)\Gamma_j(Q-\alpha_1-\alpha_2+\alpha_s)}\times\\ &\frac{\Gamma_i(\alpha_3+\alpha_t-\alpha_2)\Gamma_i(Q-\alpha_t-\alpha_1+\alpha_4)\Gamma_i(\alpha_1+\alpha_4-\alpha_t)\Gamma_i(\alpha_t+\alpha_4-\alpha_1)}{\Gamma_j(\alpha_s+\alpha_1-\alpha_2)\Gamma_j(Q-\alpha_s-\alpha_t-\alpha_4)\Gamma_j(\alpha_3+\alpha_4-\alpha_s)\Gamma_j(\alpha_s+\alpha_4-\alpha_3)}\times\\ &\frac{\Gamma_i(\alpha_t+\alpha_1+\alpha_4-Q)\Gamma_{\rm NS}(2Q-2\alpha_s)\Gamma_{\rm NS}(2\alpha_s)}{\Gamma_j(\alpha_s+\alpha_3+\alpha_4-Q)\Gamma_{\rm NS}(Q-2\alpha_t)\Gamma_{\rm NS}(2\alpha_t-Q)}\cdot\frac{1}{i}\int_{-i\infty}^{i\infty}d\tau J_{\alpha_s,\alpha_t}\Big[\alpha_{3241}\Big]_j^i,\end{aligned} \tag{24}$$

$$\begin{aligned}J_{\alpha_s,\alpha_t}\Big[\alpha_{3241}\Big]_1^1=&\frac{S_{\rm NS}(Q+\tau-\alpha_1)S_{\rm NS}(\tau+\alpha_4+\alpha_2-\alpha_3)}{S_{\rm NS}(Q+\tau+\alpha_4-\alpha_t)S_{\rm NS}(\tau+\alpha_4+\alpha_t)}\times\\ &\frac{S_{\rm NS}(\tau+\alpha_1)S_{\rm NS}(\tau+\alpha_4+\alpha_2+\alpha_3-Q)}{S_{\rm NS}(Q+\tau+\alpha_2-\alpha_s)S_{\rm NS}(\tau+\alpha_2+\alpha_s)}+({\rm NS}\leftrightarrow{\rm R}),\end{aligned} \tag{25}$$

$$
\begin{aligned}
J_{\alpha_s,\alpha_t}\left[\alpha_{3241}\right]_2^1 = & \frac{S_{\rm NS}(Q+\tau-\alpha_1)S_{\rm NS}(\tau+\alpha_4+\alpha_2-\alpha_3}{S_{\rm NS}(Q+\tau+\alpha_4-\alpha_t)S_{\rm NS}(\tau+\alpha_4+\alpha_t)}\times \\
& \frac{S_{\rm NS}(\tau+\alpha_1)S_{\rm NS}(\tau+\alpha_4+\alpha_2+\alpha_3-Q)}{S_{\rm R}(Q+\tau+\alpha_2-\alpha_s)S_{\rm R}(\tau+\alpha_2+\alpha_s)} - ({\rm NS}\leftrightarrow {\rm R}),
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
J_{\alpha_s,\alpha_t}\left[\alpha_{3241}\right]_1^2 = & \frac{S_{\rm NS}(Q+\tau-\alpha_1)S_{\rm NS}(\tau+\alpha_4+\alpha_2-\alpha_3}{S_{\rm R}(Q+\tau+\alpha_4-\alpha_t)S_{\rm R}(\tau+\alpha_4+\alpha_t)}\times \\
& \frac{S_{\rm NS}(\tau+\alpha_1)S_{\rm NS}(\tau+\alpha_4+\alpha_2+\alpha_3-Q)}{S_{\rm NS}(Q+\tau+\alpha_2-\alpha_s)S_{\rm NS}(\tau+\alpha_2+\alpha_s)} - ({\rm NS}\leftrightarrow {\rm R}),
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
J_{\alpha_s,\alpha_t}\left[\alpha_{3241}\right]_2^2 = & \frac{S_{\rm NS}(Q+\tau-\alpha_1)S_{\rm NS}(\tau+\alpha_4+\alpha_2-\alpha_3}{S_{\rm R}(Q+\tau+\alpha_4-\alpha_t)S_{\rm R}(\tau+\alpha_4+\alpha_t)}\times \\
& \frac{S_{\rm NS}(\tau+\alpha_1)S_{\rm NS}(\tau+\alpha_4+\alpha_2+\alpha_3-Q)}{S_{\rm R}(Q+\tau+\alpha_2-\alpha_s)S_{\rm R}(\tau+\alpha_2+\alpha_s)} + ({\rm NS}\leftrightarrow {\rm R}).
\end{aligned} \tag{28}
$$

NS Sector Fusion Matrix. Here we compute the elements of the fusion matrix with one of the intermediate states set to the vacuum and show that they give rise to the structure constants. For this purpose we will use the formula [6] and properties of the supersymmetric Barnes functions, reviewed in next Section:

$$
\begin{aligned}
\sum_{\nu=0,1}(-1)^{\nu(1+\sum_i(\nu_i+\mu_i))/2}\int\frac{dx}{i}\prod_{i=1}^{3}S_{\nu+\nu_i}(x+a_i)S_{1+\nu+\mu_i}(-x+b_i) = \\
= 2\prod_{i,j=1}S_{\nu_i+\mu_j}(a_i+b_j),
\end{aligned} \tag{29}
$$

$$
\sum_i(\nu_i+\mu_i) = 1 \bmod 2 \quad \text{and} \quad \sum_i(a_i+b_i) = Q. \tag{30}
$$

Recall that structure constants in the NS sector are given by Eqs. (19) and (20) and fusion matrix by (24).

We show that

$$
F_{0,\alpha_t}\left[\alpha_{3131}\right]_1^1 = C_{\rm NS}(\alpha_t,\alpha_1,\alpha_3)\frac{W_{\rm NS}(Q)W_{\rm NS}(\alpha_t)}{\pi W_{\rm NS}(Q-\alpha_1)W_{\rm NS}(Q-\alpha_3)}, \tag{31}
$$

$$
F_{0,\alpha_t}\left[\alpha_{3131}\right]_1^2 = \tilde{C}_{\rm NS}(\alpha_t,\alpha_1,\alpha_3)\frac{W_{\rm NS}(Q)W_{\rm NS}(\alpha_t)}{\pi W_{\rm NS}(Q-\alpha_1)W_{\rm NS}(Q-\alpha_3)}, \tag{32}
$$

$$
\tilde{F}_{\alpha_s,0}\left[\alpha_{2211}\right]_1^1 = \lim_{\varepsilon\to 0}\varepsilon^2 F_{\alpha_s,\varepsilon}\left[\alpha_{2211}\right]_1^1 = \frac{4}{\pi C_{\rm NS}(\alpha_s,\alpha_2,\alpha_1)}\cdot\frac{W_{\rm NS}(0)W_{\rm NS}(Q-\alpha_s)}{W_{\rm NS}(\alpha_1)W_{\rm NS}(\alpha_2)}, \tag{33}
$$

$$
\tilde{F}_{\alpha_s,0}\left[\alpha_{2211}\right]_2^1 = \lim_{\varepsilon\to 0}\varepsilon^2 F_{\alpha_s,\varepsilon}\left[\alpha_{2211}\right]_2^1 = \frac{4}{\pi \tilde{C}_{\rm NS}(\alpha_s,\alpha_2,\alpha_1)}\cdot\frac{W_{\rm NS}(0)W_{\rm NS}(Q-\alpha_s)}{W_{\rm NS}(\alpha_1)W_{\rm NS}(\alpha_2)}. \tag{34}
$$

Note also the relations

$$
F_{0,\alpha_s}\left[\alpha_{1212}\right]_1^1\tilde{F}_{\alpha_s,0}\left[\alpha_{2211}\right]_1^1 = \frac{S(0)S(\alpha_s)}{\pi^2 S(\alpha_1)S(\alpha_2)}, \tag{35}
$$

$$
F_{0,\alpha_s}\left[\alpha_{1212}\right]_1^2\tilde{F}_{\alpha_s,0}\left[\alpha_{2211}\right]_2^1 = \frac{S(0)S(\alpha_s)}{\pi^2 S(\alpha_1)S(\alpha_2)}, \tag{36}
$$

where $S(\alpha) = \sin\pi b(\alpha-Q/2)\sin(\pi/b)(\alpha-Q/2)$. We see that the relations (31)–(36) indeed have the structure of the Eqs. (2), (4) and (5).

Supersymmetric Barnes Functions. The function $\Gamma_b(x)$ is a close relative of the double Gamma function. It can be defined by means of the integral representation

$$\log\Gamma_b(x)=\int_0^\infty \frac{dt}{t}\left(\frac{e^{-xt}-e^{-Qt/2}}{(1-e^{-bt})(1-e^{-t/b})}-\frac{(Q-2x)^2}{8e^t}-\frac{Q-2x}{t}\right). \tag{37}$$

$\Gamma_b(x)$ satisfies the functional equation $\Gamma_b(x+b)=\sqrt{2\pi}b^{bx-\frac{1}{2}}\Gamma^{-1}(bx)\Gamma_b(x)$. In the super Liouville theory are important the functions

$$\Gamma_1(x)\equiv\Gamma_{\mathrm{NS}}(x)=\Gamma_b\left(\frac{x}{2}\right)\Gamma_b\left(\frac{x+Q}{2}\right), \tag{38}$$

$$\Gamma_0(x)\equiv\Gamma_{\mathrm{R}}(x)=\Gamma_b\left(\frac{x+b}{2}\right)\Gamma_b\left(\frac{x+b^{-1}}{2}\right). \tag{39}$$

They have the properties:

$$\frac{\Gamma_{\mathrm{A}}(2\alpha)}{\Gamma_{\mathrm{A}}(2\alpha-Q)}=W_{\mathrm{A}}(\alpha)\lambda^{\frac{Q-2\alpha}{2b}}, \quad \mathrm{A}=\mathrm{NS},\mathrm{R}, \tag{40}$$

where $W_{\mathrm{NS}}(\alpha)$, $W_{\mathrm{R}}(\alpha)$ are defined in (16) and (17), and $\lambda=\pi\mu\gamma\left(\frac{bQ}{2}\right)b^{1-b^2}$. The structure constants in the super Liouville theory are defined in terms of the functions:

$$\Upsilon_1(x)=\frac{1}{\Gamma_{\mathrm{NS}}(x)\Gamma_{\mathrm{NS}}(Q-x)}, \quad \Upsilon_0(x)=\frac{1}{\Gamma_{\mathrm{R}}(x)\Gamma_{\mathrm{R}}(Q-x)}. \tag{41}$$

They have the properties

$$\frac{\Upsilon_{\mathrm{A}}(2x)}{\Upsilon_{\mathrm{A}}(2x-Q)}=\mathcal{G}_A(x)\lambda^{-\frac{Q-2x}{b}}, \quad \mathrm{A}=\mathrm{NS},\mathrm{R}, \tag{42}$$

where $\mathcal{G}_{\mathrm{NS}}(x)$ and $\mathcal{G}_{\mathrm{R}}(x)$ are defined in (18). To write fusion matrix, we need also the functions:

$$S_1(x)\equiv S_{\mathrm{NS}}(x)=\frac{\Gamma_{\mathrm{NS}}(x)}{\Gamma_{\mathrm{NS}}(Q-x)}, \quad S_0(x)\equiv S_{\mathrm{R}}(x)=\frac{\Gamma_{\mathrm{R}}(x)}{\Gamma_{\mathrm{R}}(Q-x)}. \tag{43}$$

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