▸ ANAHIT CHUBARYAN, ARTUR KHAMISYAN, *Application of Kalmar's proof of deducibility in two valued propositional logic for many valued logic.*
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We focus on the problem of constructing of some standard Hilbert style proof systems for any version of many valued propositional logic. The generalization of well-known Kalmar's proof of deducibility for two valued tautologies inside classical propositional logic [1] gives us a possibility to suggest some method for defining of two types axiomatic systems for any version of 3-valued logic, completeness of which is easy proved direct, without of loading into two valued logic.

First of constructed system bases on the logic with one designated value and *conjunction, disjunction, implication*, defined by Gödel, and *negation*, defined by permuting of truth values cyclically. For every formula A, B, C of 3-valued logic, each σ_1, σ_2 from the set $\{0, 1/2, 1\}$ and $* \in \{\&, \vee, \supset\}$, the following formulas are axioms schemes

1. $A \supset (B \supset A)$
2. $(A \supset B) \supset ((A \supset (B \supset C)) \supset (A \supset C))$
3. $A^{\sigma_1} \supset (B^{\sigma_2} \supset (A * B)^{\varphi_*(A,B,\sigma_1,\sigma_2)})$
4. $A^{\sigma} \supset (\neg A)^{\bar{\sigma}}$
5. $(A \supset B) \supset ((\bar{A} \supset B) \supset ((\bar{\bar{A}} \supset B) \supset B))$, where

$$\varphi_{\supset}(A, B, \sigma_1, \sigma_2) = (\sigma_1 \supset \sigma_2)\&(\neg(A \vee \bar{A}) \vee (\bar{\bar{B}} \supset B)) \vee (\neg(A \vee \bar{\bar{A}})\&\neg(B \vee \bar{\bar{B}})),$$

$$\varphi_{\vee}(A, B, \sigma_1, \sigma_2) = (\sigma_1 \vee \sigma_2) \vee (A \supset \bar{A})\&\neg(\bar{B} \vee \bar{\bar{B}})) \vee (\neg(\bar{A} \vee \bar{\bar{A}})\&(B \supset \bar{B})),$$

$$\varphi_{\&}(A, B, \sigma_1, \sigma_2) = (\sigma_1\&\sigma_2) \vee ((A\&\bar{\bar{A}}) \vee (B\&\bar{B})) \vee ((A\&\bar{A}) \vee (B\&\bar{\bar{B}})$$

and for $\delta = \frac{i}{2}$ $(0 \leqslant i \leqslant 2)$ A^{δ} is A with $2 - i$ negations. Inference rule is *modus ponens.*

Note that axioms 3.-4. are generalizations of formulas, using in Kalmar's proof of deducibility for two valued tautologies, therefore the completeness of this system is proved very easy. This method i) can be base for direct proving of completeness for all well-known axiomatic systems of k-valued $(k \geqslant 3)$ logics and may be for fuzzy logic also, ii) can be base for constructing of new Hilbert-style axiomatic systems for all mentioned logics.

Second system obtained from first one by some restrictions, which allow to obtain the same by order bounds of main proof complexity characteristics for large sets of k-tautologies.

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[1] E. MENDELSON, *Introduction to Mathematical Logic*, ***Van Nostrand, Princeton***, 1975.