Bigroup of Operations

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By analogy of bilattices [1, 2, 3] we consider the concepts of a bisemigroup, a bimonoid, a De Morgan bisemigroup and a bigroup.

A bisemigroup is an algebra $Q(\cdot,\circ)$ equipped with two binary associative operations $\cdot$ and $\circ$. If both of these operations have an identity element, then the bisemigroup is called a bimonoid. A commutative bisemigroup is a bisemigroup in which both operations are commutative. A bisemilattice is a commutative bisemigroup in which both operations are idempotent. In any bisemilattice $Q(\cdot,\circ)$, the binary operations determine two partial orders $\leqslant_1$ and $\leqslant_2$. A bisemilattice is called a bilattice, if the partial orders $\leqslant_1$ and $\leqslant_2$ are lattice orders. Since every lattice order is characterized by two binary operations, every bilattice is a binary algebra with four operations and corresponding identities. A De Morgan bisemigroup is an algebra $Q(\cdot,\circ,^-,0,1)$ such that $Q(\cdot,\circ)$ is a bimonoid with identity elements 0 (for operation $\cdot$), 1 (for the operation $\circ$) and such that the identities

$\overline{\overline{x}} = x,$
$\overline{x\cdot y} = \overline{x}\circ\overline{y},$
$\overline{x\circ y} = \overline{x}\cdot\overline{y},$
$x\circ 0 = 0\circ x = 0,$
$x\cdot 1 = 1\cdot x = 1$

hold. A De Morgan bisemigroup $Q(\cdot,\circ,^-,0,1)$ is a De Morgan algebra if $Q(\cdot,\circ)$ is a distributive lattice .

Let Q be an arbitrary non-empty set, let $O_p^{(n)}Q$ be a set of all n-ary operations on Q, and:

$$O_pQ = \bigcup_n O_p^{(n)}Q;$$

A bimonoid $Q(\cdot,\circ)$ with identity elements 0 (for operation $\cdot$) and 1 (for operation $\circ$) is called a bigroup, if

$x\circ 0 = 0\circ x = 0,$
$x\cdot 1 = 1\cdot x = 1,$

and the following conditions are valid:

a) $Q\setminus\{1\}$ is a group with an identity element 0 under the multiplication $\cdot$;

b) $Q\backslash\{0\}$ is a group with an identity element 1 under the multiplication $\circ$;

A bigroup of order > 3 is called non-trivial.

The set $O_p^{(2)}Q$ of all binary operations on the set Q is a bimonoid under the following operations:

$$f \cdot g(x,\, y) = f(x,\, g(x,\, y)), \tag{1}$$

$$f \circ g(x,\, y) = f(g(x,\, y),\, y), \tag{2}$$

in which the identity elements are the identical operations δ_2^2 and δ_2^1,where $\delta_2^1(x,\, y) = x$, and $\delta_2^2(x,\, y) = y$ for all $x, y \in Q$. Any subset $S \subseteq O_p^{(2)}Q$ which is closed under these two operations is called a bisemigroup of operations (on the set Q). The bisemigroup of operations (on the set Q) is called a bimonoid of operations (on the set Q) if it contains the identical operations δ_2^1 and δ_2^2.

The bimonoid S of operations (on the set Q) is a bigroup, if both of the following conditions are valid:

c) $S \setminus \left\{\delta_2^1\right\}$ is a group with an identity element δ_2^2 under the multiplication (1) ;

d) $S \setminus \left\{\delta_2^2\right\}$ is a group with an identity element δ_2^1 under the multiplication (2) ;

Such bigroup is called a bigroup of operations (on the set Q).

We characterize bigroups of operations in the category of second order algebras introduced in [4].

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