On the solvability of pseudodifferential equations in spaces with quasihomogeneous norm

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Let R^n be the Euclidean space with points $x = (x_1, x_2, \ldots, x_n)$, $r = (r_1, r_2, \ldots, r_n)$ a vector with positive components, $1/r* = 1/n \sum_1^n 1/r_j$, and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$, where $\lambda_j = r^*/r_j$, $j = 1, 2, \ldots, n$. By $\rho(x)$ we denote the function, positive for $x \neq 0$, defined implicitly by the equality

$$\sum_{i=1}^{n} x_i^2 \rho^{-2\lambda_i} = 1.$$

It is natural (see [4]) to denote the completion of $C_0^\infty(R^n)$ in the norm

$$\|f\| = \left\| F^{-1}\left(\rho^{r^*}(\xi) F\varphi(\xi)\right)\right\|_p, \quad 1 < p < \infty,$$

by the simbol $\dot{w}_p^r$ (see also [1] and [2]) and called space with quasihomogeneous norm, or space of anisotropic potentials. If $r^* < n/p$, then $\dot{w}_p^r$ is the space of functions representable with anisotropic potentials. When $r^* \geq n/p$ the space $\dot{w}_p^r$ is no longer a function space; its elements are classes in which functions that differ by a corresponding polynomial are identified (see [3], [4]). When $r_1 = r_2 = \cdots = r_n = 0$ we set $\dot{w}_p^r = L_p(R^n)$. The space $\dot{w}_p^{-r}$ is defined as the dual of $\dot{w}_p^r$.

Let us suppose that the function $K(\xi)$ is λ-homogeneous of degree s, $-\infty < s < \infty$, i.e. for every $t > 0$ and arbitrary $\xi \neq 0$, $K(t^\lambda \xi) \equiv K(t^{\lambda_1}\xi_1, \ldots, t^{\lambda_n}\xi_n) = t^s K(\xi)$. As shown in [4] continuous for $\xi \neq 0$ λ-homogeneous $K(\xi)$ function is the symbol of a bounded operator

$$K : \dot{w}_2^r \longrightarrow \dot{w}_2^{\kappa r},$$

where $\kappa = 1 - s/r^*$ and $\kappa r = (\kappa r_1, \ldots, \kappa r_n)$.

Let $\Psi = (\Psi_{jk})$, $j = 0, 1, \ldots, M-1$; $k = 0, 1, \ldots, N-1$, is the matrix of ΨDO Ψ_{jk} with symbol $\Psi_{jk}(\xi)$ continuous for $\xi \neq 0$ and λ-homogeneous of degree $\alpha_j - \beta_k$:

$$\Psi_{jk} = (t^{\lambda_1}\xi_1, \ldots, t^{\lambda_n}\xi_n) = t^{\alpha_j - \beta_k}\Psi_{jk}(\xi), \quad t > 0, \ -\infty < \alpha_j, \beta_k < \infty,$$

$u = (u_0, u_1, \ldots, u_{N-1})$, $f = (f_0, f_1, \ldots, f_{M-1})$.

Now we consider the solvability of the system of equations

$$\Psi u = f$$

in $\dot{w}_2^r$. If $M = N = 1$ the following theorem was proved in [5].

Theorem 1. ΨDO

$$\Psi : \prod_{k=0}^{N-1} \dot{w}_2^{(1-\beta_k/r^*)r} \longrightarrow \prod_{j=0}^{M-1} \dot{w}_2^{(1-\alpha_j/r^*)r}$$

is bounded and left-invertible (right- invertible) if and only if $\operatorname{rank}(\Psi_{jk}(\xi)) = N$ *(respectively, M) for all* $\xi \neq 0$.

References

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