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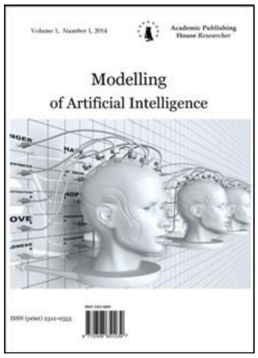

UDC 004

Monte Carlo Method for Geometric Average Options on Several Futures

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Abstract

In this paper we use Monte-Carlo method to price Asian options based on futures contracts with multiple maturities in the averaging period. We first derive a closed form solution for Geometric average option based on Andersen's commodity model and then use the result as a control variate. Our method eliminates the need for ad-hoc approximations commonly used in practice when pricing Asian options based on multiple future contract months.

Keywords: asian option, future price, Monte Carlo method, geometric average options.

1. Introduction

The paper continues the investigations in (Kechejian, Ohanyan, 2012) and (Kechejian et al., 2015). We consider Asian options based on several futures and fixed strike prices. The payoff of this kind of contracts have the following form

$$\Psi\big(F(t)\big) = \left(\frac{1}{T_n - t_0}\left(\int_{t_0}^{T_1} F(u, T_1')du + \int_{T_1}^{T_2} F(u, T_2')du + \dots + \int_{T_{n-1}}^{T_n} F(u, T_n')du\right) - K\right)^+ \quad (1.1)$$

where t_0 is the initial point of the averaging period, and T_n is the endpoint. Each future contract expiry point is denoted by T_i' Note that the future contract prices are not necessary used in the averaging period up to their maturity. Asian options are not easy to value by direct methods and our case is not an exception. Most techniques use approximations and several are based on equivalent geometric average options, for example Curran's methods (Curran, 1994). Some use approximation methods based on moments, for example German-Yor method using Laplace transformation (Geman, Yor, 1993). Here we implement the technique described in (Zhang, 2009).

To this end we first evaluate the price of the geometric average option.

For the geometric average option the payoff is

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$$\Psi_g\big(F(t)\big) = \left(\exp\left(\frac{1}{T_n - t_n - t_0}\left(\int_{t_0}^{T_1} \ln(F(u,T_1')du + \int_{T_1}^{T_2} \ln(F(u,T_2')du + \cdots + \int_{T_{n-1}}^{T_n - t_n} \ln(F(u,T_n')du\right)\right) - K\right)^+ \quad (1.2)$$

In this paper we use more appropriate formula, where instead of integrals we use sums, taking discrete averaging, since all Asian options in practice are structured using discrete averaging.

Moments of geometric mean option on futures

In this section geometric average options prices computations are done for the analogs of formulas (1.1) and (1.2) with discrete averaging. The price formulas are

$$\Psi(F) = E\left[\frac{1}{N}\left(\sum_{i=1}^{m_1} F\big(t_{i,1},T_1'\big) + \sum_{i=1}^{m_2} F\big(t_{i,2},T_2'\big) + \ \ldots\ + \sum_{i=1}^{m_n} F\big(t_{i,n},T_n'\big)\right) - K\right]^+ \quad (2.1)$$

$$\Psi_g(F) = E\left[\exp\left\{\frac{1}{N}\left(\sum_{i=1}^{m_1} \ln F\big(t_{i,1},T_2'\big) + \sum_{i=1}^{m_2} \ln F\big(t_{i,2},T_2'\big) + \ \ldots\ + \sum_{i=1}^{m_n} \ln F\big(t_{i,n},T_n'\big)\right)\right\} - K\right]^+ \quad (2.2)$$

where $N = \sum_{j=1}^{n} m_j$. In the sequence $\{t_{j,k}\}$, j stands for days, and k for the series of future contracts. For example, the third pricing day of the second future contract (which is expiring at T2) we have the

price $F(t_{3,2},T_2)$.

For computations we use the model described by Anderson in (Andersen, 2008). Here future prices are distributed log-normally and described by the following equation.

$$d\ln\big(F(t,T)\big) = -|\sigma(t,T)|^2 dt + \sigma(t,T)' dW(u)$$

Here the $\sigma(t,T) = (\sigma_1(t,T), \sigma_2(t,T))$ is taken as following. $\sigma_1(t,T) = h_1 e^{b(T)-k(T-t)} + h_\infty e^{a(T)}$; $\sigma_2(t,T) = h_2 e^{b(T)-k(T-t)}$.

Functions a(T) and b(T) are seasonality adjustments, parameter k represents the mean-reversion speed, and k, h_1, h_2, h_∞ are constants. The quantities $\sigma_1(t,T)$ and $\sigma_2(t,T)$ are responsible for the long run and short run volatilities respectively.

So for this particular case we can further write (Andersen, 2008):

$$\ln\big(F(t,T)\big) = \ln\big(F(0,T)\big) + e^{a(T)}\left(z_1(t)e^{-k(T-t)+d(T)} + z_2(t)\right) - \frac{e^{2a(T)}}{4k} \times$$
$$\times \left[e^{2d(T)-2kT}\left(e^{2kt}-1\right)\left(h_1^2+h_2^2\right) + 4h_1 h_\infty e^{d(T)-kT}\left(e^{kt}-1\right) + 2h_\infty^2 tk\right] \quad (2.3)$$

where $(T) = b(T) - a(T)$;
$dz_1(t) = -kz_1(t)dt + h_1 dW_1(t) + h_2 dW_2(t)$; $dz_1(t) = h_\infty dW_1(t)$:
with $z_1(0) = z_2(0) = 0$ and $\ln\big(F(0,T)\big)$ is given.

Here the future contract is driven by two independent Wiener processes (see Andersen, 2008: 29-31). We are able to compute the moments for the sum of logarithms of future prices. This is due to the fact that both $z_1(t)$ and $z_2(t)$ are Gaussian processes (see also (Kechejian et al., 2016):

$$\sum_{s=1}^{n}\sum_{i=1}^{m_s} \ln\left(F\big(t_{i,s},T_s\big)\right) \quad (2.4)$$

Has normal distribution with parameters μ, and $\sigma^2$where the parameters have the following form.

$$\mu = \sum_{s=1}^{n} m_s \ln F(0,T_s) - (h_1^2 + h_2^2)\left(\sum_{s=1}^{n}\sum_{i=1}^{m_s}\frac{e^{2a(T_s)}}{4k}e^{2d(T_s)-kT_s}\left(e^{2kt_{i,s}}-1\right)\right) - 2h_\infty^2\left(\sum_{s=1}^{n}\sum_{i=1}^{m_s}\frac{e^{2a(T_s)}}{4k}kt_{i,s}\right) \quad (2.5)$$

and

$$\sigma^2 = h_2^2\left(\sum_{s=1}^{n} e^{2(a(T_s)-kT_s+d(T_s))}\left(\sum_{i=1}^{m_s}\frac{1}{2k}\left(e^{2kt_{i,s}}-1\right)+\sum_{i=1}^{m_s}(m_s-i)\frac{1}{k}\left(e^{2kt_{i.s}}-1\right)\right) + \sum_{l=1}^{n-1} e^{a(T_l)-kT_l+d(T_l)}\sum_{s=l}^{n} e^{a(T_s)-kT_s+d(T_s)}m_s\sum_{i=1}^{m_s}\frac{1}{k}\left(e^{2kt_{i.l}}-1\right)\right) + \left(\sum_{s=1}^{n} e^{2a(T_s)}\sum_{i=1}^{m_s-1}\int_0^{t_{i,s}}\left(e^{-kT_s+d(T_s)+kv}h_1+h_\infty\right)^2 dv + 2\sum_{s=1}^{n} e^{2a(T_s)}\sum_{i=1}^{m_n}(m_s-i)\int_0^{t_{i,s}}\left(e^{-kT_s+d(T_s)+kv}h_1+h_\infty\right)^2 dv\right) \quad (2.6)$$

$$+\left(2\sum_{l=1}^{n-1} e^{2a(T_l)}\sum_{s=l}^{n} e^{2a(T_s)}m_s\sum_{i=1}^{m_s}\int_0^{t_{i,l}}\left(e^{-kT_l+d(T_l)+kv}h_1+h_\infty\right)\left(e^{-kT_s+d(T_s)+kv}h_1 + h_\infty\right)dv\right)$$

So the price of the geometric average option is given by

$$E\left(\Psi_{geom}\right) = -K\left(1-\Phi\left(\frac{n\ln K-\mu}{\sigma}\right)\right)+e^{\frac{1\sigma^2}{2n^2}+\frac{\mu}{n}}\left(1-\Phi\left(\frac{n\ln K-\mu}{\sigma}-\frac{\sigma}{n}\right)\right) \quad (2.7)$$

where $\Phi(\cdot)$ is the distribution function of standard normal random variable, and σ_y is time scaled standard deviation.

Monte Carlo with control variate

We use the results from previous section to set-up Monte Carlo method with control variate to evaluate arithmetic Asian options. For this technique see for example (Zhang, 2009) and more detailed explanation can be seen in (Glasserman, 2003).

Briefly we do the following. We first calibrate the model to find best fitting parameters k, h_1, h_2, h_∞. To accomplish the latter we use the least squares method. Namely we minimize the following function

$$G(k,h_1,h_2,h_\infty) = \sum_{i=1}^{m} g_i^2 \quad (3.1)$$

where g_i, $i = 1,\dots,m$ is

$$g_i = \frac{e^{2kT^i}}{2k(T_2^i-T_1)}\left(e^{2kT_2^i}-e^{kT_1}\right)(h_1^2+h_2^2)+\frac{e^{-kT^i}}{k(T_2^i-T_1)}h_1h_\infty\left(e^{kT_3^i}-e^{kT_1}\right)+h_\infty^2 - \sigma_{term}^2\left(T^i\right) \quad (3.2)$$

Minimizing G we find the best fitting parameters. By differentiation we get the following system of four equations with unknown variables k, h_1, h_2, h_∞

$$\begin{cases} \frac{\partial G}{\partial h_1} = 4\sum_{i=1}^{m} g_i \left[h_1 \frac{e^{2kT^i}}{2k(T_2^i - T_1)} \left(e^{2kT_2^i} - e^{2kT_1} \right) + h_\infty \frac{e^{kT^i}}{k(T_2^i - T_1)} \left(e^{kT_2^i} - e^{kT_1} \right) \right] = 0 \\ \frac{\partial G}{\partial h_2} = 4\sum_{i=1}^{m} g_i h_2 \frac{e^{2kT^i}}{2k(T_2^i - T_1)} \left(e^{2kT_2^i} - e^{2kT_1} \right) = 0 \\ \frac{\partial G}{\partial h_\infty} = 4\sum_{i=1}^{m} g_i \left[h_1 \frac{e^{kT^i}}{k(T_2^i - T_1)} \left(e^{kT_2^i} - e^{kT_1} \right) + h_\infty \right] = 0 \\ \frac{\partial G}{\partial k} = 4\sum_{i=1}^{m} g_i \frac{1}{2k^2(T_2^i - T_1)} [(h_1^2 + h_2^2) e^{-2kT^i} \times \\ \times \left[\left(e^{2kT_2^i} - e^{2kT_1} \right) (-2T^i k - 1) + 2k \left(T_2^i e^{2kT_2^i} - T_1 e^{2kT_1} \right) \right] + \\ +4h_1 h_\infty e^{-kT^i} \left[\left(e^{2kT_2^i} - e^{2kT_1} \right) (-2T^i k - 1) + 2k \left(T_2^i e^{2kT_2^i} - T_1 e^{2kT_1} \right) \right]] = 0 \end{cases}$$

However we don't solve this directly.

Next we generate future prices based on Andersen model. Then we compute the geometric and arithmetic mean option (A_j, G_j), using the same strike price , just simulating it. Then we compute the mean based on the formulas (2.6) and (2.7), and finally compute the following

$$\theta = A_j - G_j + E(\Psi_{geom}) \tag{3.4}$$

2. Results

Using the data on future contract and corresponding option price for the period from June 2016 up to December 2024, we did the model calibration and got the following values for k, h_1, h_2, h_∞.

h_1	0,170501
h_2	0,23932
h_infty	0,2
kappa	-0,12673

And so obtain the following mean with Monte Carlo method 9,181471, for the strike 30. This is obtained for Asian options of 3 nearest months.

Contract	Futures	38,5
Jun-16	38,94	44,17
Jul-16	39,78	44,55
Aug-16	40,35	44,53
Sep-16	40,8	44,23
Oct-16	41,15	43,8
Nov-16	41,44	43,47
Dec-16	41,7	43,04
Jan-17	41,93	42,38
Feb-17	42,16	41,39
Mar-17	42,38	40,78
Apr-17	42,61	40,19
May-17	42,83	39,45
Jun-17	43,05	38,97

Jul-17	43,23	38,17
Aug-17	43,4	37,61
Sep-17	43,58	37,09
Oct-17	43,77	36,39
Nov-17	43,98	35,91
Dec-17	44,18	35,49
Jan-18	44,3	35,06
Feb-18	44,43	34,73
Mar-18	44,58	34,62
Apr-18	44,74	34,02
May-18	44,88	33,56
Jun-18	45,04	33,12
Jul-18	45,17	32,5
Aug-18	45,31	32,13
Sep-18	45,47	31,93
Oct-18	45,63	31,88
Nov-18	45,82	31,99
Dec-18	46,01	30,69
Jan-19	46,11	30,35
Feb-19	46,23	30,09
Mar-19	46,36	29,91
Apr-19	46,49	29,76
Jun-19	46,8	29,41
Dec-19	47,63	28,68
Jan-20	47,7	28,36
Jun-20	48,28	27,14
Dec-20	48,93	26,02
Jun-21	49,46	25,73
Dec-21	49,91	25,63
Jun-22	50,28	23,79
Dec-22	50,65	22,18
Jun-23	50,9	21,01
Dec-23	51,15	20,08
Jun-24	51,32	20
Dec-24	51,48	20,01

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УДК 004

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Аннотация. В этой статье мы применяем метод Монте-Карло для расчёта цен Азиатских опционов для фючерсных контрактов с несколькими промежутками времени в периоде усреднения. Сначало мы получаем формулу для расчёта цен опционов с геометрическим средним основываясь на модели для товарных фючерсов, представленной в работе Андерсона, после чего используем результаты в качестве контрольных переменных в методе Монте-Карло. Представленный метод устраняет необходимость использования приближений для каждого отдельного случая, часто употребляемого в практике при оценки азиатских опционов с несколькими фючерсами.

Ключевые слова: азиатский опцион, фьючерс цена, метод Монте-Карло, средние геометрические параметры.

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