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Threshold Properties of Free Electron Lasers without Inversion

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Abstract—It is shown that the possibility to create the free electron laser without inversion (FELWI) has a threshold behavior on the field of intensity of amplified wave. In the collective approach, the description of threshold conditions is given. It is shown that the threshold of observation of amplification without inversion is sufficiently high, which hampers essentially the possibility of experimental realization of the FELWI.

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1. INTRODUCTION

The idea to create the free electron lasers without an inversion (FELWI) was first proposed in [1], and then developed and improved in [2–5]. Specific realizations of the FELWI were proposed and considered in [6, 7]. One of the main points of the FELWI realization schemes is a proposal to use the non-collinear propagations of the electron beam and the amplified radiation. In the conventional free electron lasers (FEL) and the strophotrons these schemes are well known and have been discussed for a long time [8–27]. As applied to the FELWI with two undulators, the main idea is that at the non-collinear interaction of laser and electron beams after first undulator, there is a scattering of electrons over transversal velocities and hence of angles, and this scattering is directly related with the increase in the electron energy. Therefore, the selection of electrons over directions in the undulator gap is equivalent to the selection over the energies. In principle, it allows one in a controlled manner to change the length of the path of electrons with different energies in the inter-undulators space and the distribution of energy at the entry into the second undulator. If the devices in the inter-undulator gap have a negative dispersion property (i. e., the faster electrons spend more time on the passage of the inter-undulator space than the slow electrons), then the integral gain coefficient (over the energies of electrons) $G(\omega)$ can be positive in almost the entire region of the frequency change of the amplified wave in the vicinity of the resonance frequency of the undulator.

This mechanism can operate only if the angular spread α resulting from the interaction of electrons with the field of undulator and amplified wave is greater than the natural velocity dispersion over directions in the electron beam $\Delta\alpha_{\text{beam}}$. Virtually, $\Delta\alpha_{\text{beam}}$ cannot be less than 10^{-4} rad. The condition $\alpha > \Delta\alpha_{\text{beam}}$ leads to the origination of the realization threshold of the FELWI either by the intensity of laser radiation or by the electron beam density.

In the present work, the threshold of origination of amplification effect without the inversion in the proposed FEL schemes to achieve this goal is evaluated. The analysis is performed within the framework of the multiparticle description covering both the Compton and the Raman amplification modes in the FEL.

2. MULTIPARTICLE DESCRIPTION

In [2–5], the model of unlimited electron and laser beams was investigated, while the real beams are limited in the longitudinal direction. Allowance for the finite widths of the latter is fundamental for lasers in which the laser and electron beams are collinear, which results in the finiteness of area of their interaction. Therefore, to take into account the finite sizes of electron and laser beams, the evaluations will be made below, in particular, the origination of the threshold in the FELWI will be analyzed. To do this, we apply the method of dispersion analysis to obtain the spatial amplification of the laser wave in the magnetostatic undulator with the non-collinear geometry.

2.1. Spatial Amplification

Consider the propagation of monoenergetic electron beam in the magnetostatic undulator. The coordinate system is chosen in such a way that the Oz axis coincides with the axis of a wiggler, and the vector potential of a wiggler field would be directed along the Oy axis. Assume, that the static magnetic field of the flat undulator $\mathbf{A}_w$ is independent on the transverse coordinates x and y and is approximated by the harmonic function

$$\mathbf{A}_w = A_w\mathbf{e}_y = \left(A_0 e^{-ik_w r} + \text{c. c.}\right)\mathbf{e}_y\,, \tag{1}$$

where $\mathbf{k}_w = \left(0,0,k_w\right)$ is the wave vector of the wiggler, c. c. denotes the complex conjugation, $\mathbf{e}_y$ is a unitary vector of y-axis. We assume that the linearly polarized wave $\mathbf{A}_L = A_L(t,x,z)\mathbf{e}_y = \mathbf{a}_+ e^{i(\mathbf{k}-\mathbf{k}_w)\mathbf{r}-i\omega t}$ propagates in the wiggler, where the vector potential is directed along the y-axis, and the wave vector is in the plane xz: $\mathbf{k} = \left(k\sin\theta, 0, k\cos\theta\right)$. We have neglected the Stokes wave $\mathbf{A}_- = \mathbf{a}_- e^{i(\mathbf{k}+\mathbf{k}_w)\mathbf{r}-i\omega t}$, which does not play a significant role at the resonance and should be taken into account only at the tuning out [28].

The classical dynamics of an electron in the total field of wiggler and laser wave $A = A_w + A_L$ is described by the following Hamiltonian:

$$H = \sqrt{m^2c^4 + c^2\left(\mathbf{P} - \frac{e}{c}\mathbf{A}\right)^2} + e\varphi = mc^2\gamma + e\varphi. \tag{2}$$

Starting from this expression, the dispersion equation was obtained in [29] for the electron beam having at the wiggler input the uniform density n_b and the velocity $u = (-u\sin\alpha, 0, u\cos\alpha)$

$$D_b(\omega^2 - \omega_+^2) = K^2\omega_b^2\gamma_0^{-3}\left(c^2k^2 - \omega^2 + \omega_b^2\gamma_0^{-1}\right). \tag{3}$$

Here, the following notation is used for the frequency

$$\omega_+^2 = \left(\mathbf{k} - \mathbf{k}_w\right)^2 c^2 + \frac{\omega_b^2}{\gamma_0}. \tag{4}$$

and the dispersion function of the electron beam

$$D_b = (\omega - \mathbf{k}\mathbf{u})^2 - \Omega_b^2, \tag{5}$$

which is related to the frequency of the beam Ω_b, where

$$\Omega_b^2 = \omega_b^2\left[1-(\mathbf{ku})^2/(kc)^2\right]/\gamma_0 . \tag{6}$$

Here, $\omega_b^2 = 4\pi e^2 n_b / m$ is the square of the Langmuir frequency of the beam of electrons and

$$K = \frac{e}{mc^2}|A_0| \tag{7}$$

is the dimensionless amplitude of the wiggler field (the undulator parameter). The total relativistic factor of electrons is defined as $\gamma_0 = \sqrt{1+2K^2} \times\left(1-u^2/c^2\right)^{-1/2}$.

The dispersion equation (3) describes 4 oscillation branches $k_\upsilon = k_\upsilon(\omega)$, namely: two beam and two laser oscillation branches. At $\omega_b = 0$ these solutions have the form $(\omega - \mathbf{ku})^2 = 0$ for beam waves and $\omega^2 = \left(\mathbf{k}-\mathbf{k}_w\right)^2 c^2$ for laser waves. Below we consider the solution of the dispersion equation (3) when the resonance conditions are

$$\omega = \omega_+ = (\mathbf{k}_0\mathbf{u}) - \Omega_b , \tag{8}$$

which correspond to the maximum increment. In this case, the laser counter-propagating wave with $\omega = -|\mathbf{k}-\mathbf{k}_w|c$ can be neglected. We seek a solution in the form $\mathbf{k} = \mathbf{k}_0 + \delta\mathbf{k}$, where $\delta\mathbf{k}$ is a small complex correction to the wave vector $\delta\mathbf{k} = \mathbf{k}' + i\mathbf{k}''$. The imaginary part $\mathbf{k}''$ determines the spatial amplification of the laser wave in the undulator.

For collective regime, when $|(\mathbf{u}\delta\mathbf{k})| \ll \Omega_b$, the dispersion equation reduces to the square equation

$$\delta k^2 + \frac{K^2}{4}\frac{k_0\Omega_b}{\gamma_0^2 u}\frac{\omega^2}{(\mathbf{k}_0\mathbf{u})^2}\left(1+\frac{\omega_b^2}{\omega\,\Omega_b\gamma_0}\right)^2 F_R(\varphi) = 0 . \tag{9}$$

Here

$$F_R(\varphi) = \frac{1}{\left[\cos(\varphi-\theta) - \dfrac{k_w}{k_0}\cos\varphi\right]\cos(\varphi+\alpha)}, \tag{10}$$

where φ is an angle between the wiggler axis and the vector $\delta\mathbf{k}$. Note that the equation is valid as long as $F_R(\varphi)$ is small. Further, we will be interested in the amplification of wave along the direction of its propagation, i. e., for the case $\varphi = \theta$. In real situations $k_w \ll k_0$, so the term with k_w/k_0 in (10) can be neglected, then the spatial increment will be equal to

$$k'' = \frac{K}{2}\frac{k_0}{\gamma_0}\frac{\omega\sqrt{\Omega_b}}{(\mathbf{k}_0\mathbf{u})^{3/2}}\left(1+\frac{\omega_b^2}{\omega\,\Omega_b\gamma_0}\right). \tag{11}$$

Under conditions $\omega_b^2/(\omega\,\Omega_b\gamma_0) \ll 1$, when the current density of beam is small, the increment (9) has the normal dependence for the Raman regime [28]: it depends on the Langmuir frequency by law $\omega_b^{1/2}$. In the case of high-current beams, when $\omega_b^2/(\omega\,\Omega_b\gamma_0) \gg 1$, the increment has the anomalous behavior $\omega_b^{3/2}$.

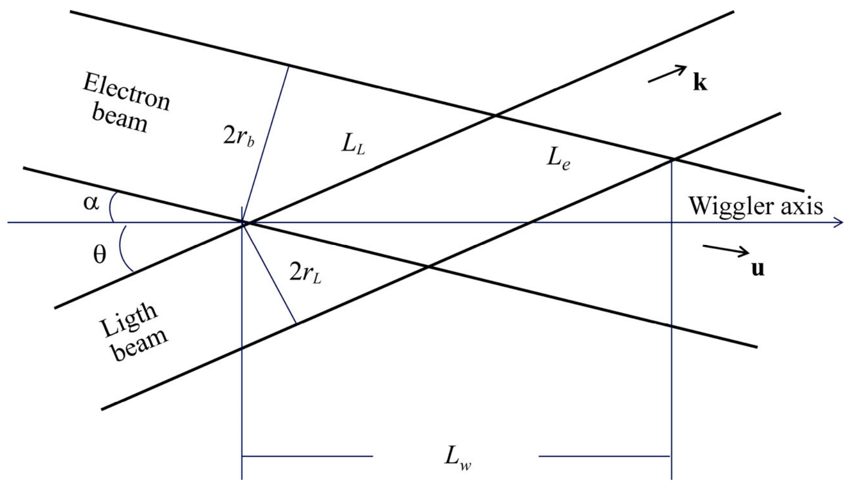
For single-particle amplification (the Thomson regime $|(\mathbf{u}\delta\mathbf{k})| \gg \Omega_b$) the dispersion equation (3) is cubic:

$$\delta k^3 + \frac{K^2}{2}\frac{k_0\Omega_b^2}{\gamma_0^2 u}\frac{\omega^2}{(\mathbf{k}_0\mathbf{u})^2}\left(1+\frac{\omega_b^2}{\omega\,\Omega_b\gamma_0}\right)^2 F_T(\varphi) = 0. \tag{12}$$

Here, $F_R(\varphi) = F_R(\varphi)/\cos(\varphi+\alpha)$. A solution of equation (12) for the imaginary part $\delta\mathbf{k}$ in the case $\varphi = \theta$ is

$$k'' = \frac{\sqrt{3}}{2}\left(\frac{K^2}{2}\right)^{1/3} k_0 \left[\frac{\omega\,\Omega_b}{\gamma_0(\mathbf{k}_0\mathbf{u})^2}\left(1+\frac{\omega_b^2}{\omega\,\Omega_b\gamma_0}\right)\right]^{2/3}. \tag{13}$$

As for the Raman regime, the normal regime with $k'' \sim \omega_b^{2/3}$ is realized in the Thomson regime for low-current beams $\omega_b^2/(\omega\,\Omega_b\gamma_0) \ll 1$ and the anomalous regime with $k'' \sim \omega_b^{4/3}$ for the high-current beams $\omega_b^2/(\omega\,\Omega_b\gamma_0) \gg 1$.



The schematic representation of propagation of electron and laser beams in the *xz*-plane of the magnetostatic undulator.

The above theoretical constructs suggest the infinite electron and light beams. In reality, both are limited in the transverse direction. For non-collinear electron and laser beams the latter circumstance results to the finite region of their interaction. The length at which the light amplification takes place in the medium of the beam of electrons is (see the figure)

$$L_L = \frac{2r_L}{\sin(\alpha+\theta)}. \tag{14}$$

Here $2r_b$ is the width of the electron beam in the *xz*-plane. In turn, the length over which the laser field does the work over the electron is

$$L_e = \frac{2r_L}{\sin(\alpha+\theta)}, \tag{15}$$

where $2r_L$ is the width of the electron beam in the *xz*-plane. The operating length of wiggler is defined by

$$L_w = L_e\cos\alpha + L_L\cos\theta \approx \frac{2(r_L + r_b)}{\sin(\alpha+\theta)}. \tag{16}$$

It makes no sense to increase the length of the wiggler more than L_w. In order to increase L_w, the quantity $\alpha+\theta$ should be decreased and the width of the electron beam $2r_b$ should be increased. The latter should be done also in virtue of the following estimates.

The gain of the laser field by the wave amplitude is equal to

$$\frac{A_{\text{out}}}{A_{\text{in}}} = \exp\left(k'' L_L\right). \tag{17}$$

As shown above, $k'' \sim \omega_b^{\nu}$. For the Thomson (single-particle) regime of instability $\nu = 2/3$ and $\nu = 4/3$, for the Raman (collective) regime of instability $\nu = 1/2$ and $\nu = 3/2$. For the cylindrical form of the beam of electrons, when the current is constant, one has the estimate $\omega_b \sim r_b^{-1}$ and therefore $k'' \sim \omega_b^{\nu} \sim r_b^{-\nu}$. Since $L_L \sim r_b$, one obtains the estimate for the gain

$$\frac{A_{\text{out}}}{A_{\text{in}}} = \exp\left(\text{const} \times r_b^{1-\nu}\right). \tag{18}$$

When $\nu < 1$, we have the monotonically increasing function on the width r_b of the beam of electron and the monotonically decreasing in the opposite case $\nu > 1$. Thus, at $\nu < 1$ a wide beam should be used and at $\nu > 1$ (super high-current electron beams) one should use the narrow beam.

2.2. Threshold for Amplification Without Inversion (AWI)

Consider here the estimations as applied to the FELWI. As mentioned above, the FELWI underlying mechanism can work only if the angular spread α resulting from the interaction of electrons with the field is greater than the natural spread $\Delta\alpha_{\text{beam}}$ of the electron beam by directions. This fact leads to the origination of the threshold for the power of laser radiation.

Regarding to its unperturbed motion $\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t$ in the *xz*-plane, the electrons of beam undergo the oscillations in this plane, with the variations of velocity equal to in the linear approximation [29]

$$\delta\mathbf{v}_{\parallel} = K^2 \frac{c^2}{\gamma_0^3} \frac{\beta_1 \mathbf{k} - \dfrac{\omega}{c^2}\beta_2 \mathbf{u}}{D_b} a e^{i\xi_0 - i\Delta_\omega t} + \text{c. c.} \tag{19}$$

Here $a = a_+ / A_0$ is the dimensionless amplitude of the laser field, $\Delta_\omega = \omega - (\mathbf{ku})$, $\xi_0 = \mathbf{k}_0 \mathbf{r}_{\parallel 0}$, $\mathbf{r}_{\parallel 0}$ is the initial coordinate in the plane *xz*. Coefficients β_1 and β_2 are equal:

$$\beta_1 = \gamma_0\left(\omega - (\mathbf{k}_0\mathbf{u})\right) - \frac{\omega_b^2 (\mathbf{k}_0\mathbf{u})}{(k_0 c)^2},$$
$$\beta_2 = \gamma_0\left(\omega - (\mathbf{k}_0\mathbf{u})\right) - \frac{\omega_b^2}{\omega}. \tag{20}$$

The equation of the trajectory (19) is obtained for a monoenergetic beam which has an infinite size, and thereby it interacts with the field endlessly. However, using this equation, one can obtain the necessary estimations, which will be carried for the most interesting case from the point of view of experiment – for the case of the single-particle regime of the gain. Assuming that an electron enters the laser field at the time instant $t = 0$ and, during the flight time $t = L_e / u$ it deflects from its original direction by an angle $\Delta\alpha$ which depends on the phase of entry into the field according to the $\cos\xi_0$ law, we obtain the maximum value

$$\Delta\alpha_{\max} \simeq K^2 \frac{c^2}{\gamma_0^2} \frac{k_0}{u^2} \sin(\alpha + \theta) \frac{e^{ik'' L_e} - 1}{k''}. \tag{21}$$

In the case of a weak amplification on the length $k'' L_e \ll 1$, the rotation angle does not depend on the angle $\alpha + \theta$, nor from the gain coefficient k'' (and hence on the beam current):

$$\Delta\alpha_{\max} \simeq \frac{2K^2 k_0 r_L}{\gamma_0^2} a. \tag{22}$$

As expected, the value $\Delta\alpha_{max}$ coincides with the estimate given in the works [30–33]. The expression (22) should be considered as the lower limit of the maximum possible deviation. The excess $\Delta\alpha_{max}$ of natural spread $\Delta\alpha_{beam}$ gives the threshold value for the amplitude of the laser field and for its intensity. Rewrite (22) in terms of the total laser power $W = \frac{c}{4}(k_0 r_L)^2 |a_+|^2$, namely by an expression defining the power threshold:

$$W > \frac{c}{8}\left(\frac{mc^2}{e}\right)^2 \frac{(\Delta\alpha_{beam})^2 \gamma_0^4}{2K^2}. \qquad (23)$$

This gives the numerical value

$$W > 1.1\times 10^9 \frac{(\Delta\alpha_{beam})^2 \gamma_0^4}{2K^2} W. \qquad (24)$$

For the following values of the parameters [7] $\gamma_0 = 15$, $K = 0.635$ rad and $\Delta\alpha_{beam} = 5\times 10^{-4}$ rad we obtain the threshold value $W > 1.8\times 10^7$ watt. The resulting threshold power exceeds the power of the laser field at which the saturation occurs. To use the laser in a linear amplification regime $\Delta\alpha_{beam}$ should be reduced. When operating at the very border of the saturation region $\sim 10^5$–10^6 W / cm^2, the formula (23) gives an estimate $\Delta\alpha_{beam} \sim 10^{-6}$ rad.

This estimate coincides with the results of [24, 30–33] obtained by an another (single-particle) approach. Note that for the stable operation of the FELWI the value $\Delta\alpha_{max}$ should exceed the natural spread of the electron beam by directions $\Delta\alpha_{beam}$ by an order of magnitude. It is doubtful that the natural angular divergence of an electron beam can be achieved in the accelerator significantly less than 10^{-6} rad.

3. CONCLUSION

The influence of the velocity spread of the electron beam by directions on the work of the FELWI is investigated. Within the multiparticle approach, the threshold value of maximal angular spread by velocities of beam electrons is obtained. It is shown that the threshold for the angular spread corresponds to the presence of a threshold by intensity (or power) of the laser field. The obtained threshold value for the power of the laser radiation (23) is an upper limit under condition of a small gain. As follows from the above estimates, the realization of the FELWI in the single-particle regime with the small gain $k''L \ll 1$ encounters the big problems that result in the practical impossibility to its realization in this regime, in the version which was considered above. There are two ways out of this situation. Firstly, the use of the Raman amplification regime. As numerical simulations have shown [7], the FELWI idea has great possibilities in this regime. Secondly, the use of strong amplification at the length $k''L \gg 1$ in the Thomson regime. In both cases there is a need for the use of the high density beams.

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