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YEREVAN STATE UNIVERSITY

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BOUNDARY INDUCED QUANTUM EFFECTS IN LINEARLY EXPANDING COSMOLOGICAL MODELS

The Casimir effect (for reviews see [1]) arises as a consequence of the modification of the spectrum for the vacuum fluctuations caused by the imposition of boundary conditions on the operator of a quantum field. As a result of that, the vacuum expectation values (VEVs) of physical observables are changed. In particular, vacuum forces arise acting on the constraining boundaries. In the present paper we consider the scalar Casimir densities and forces for the geometry of two parallel plates in background of a linearly expanding spatially flat $(D+1)$ - dimensional cosmological model. We discuss the asymptotics of the mode functions and the most important special cases corresponding to the adiabatic and conformal vacuum states. A general expression for the Hadamard function is obtained and then it is further transformed for the case of a conformally coupled scalar filed prepared in the conformal vacuum. The VEVs of the field squared and of the energy-momentum tensor and the Casimir forces for this special case are investigated.

Problem setup. As a background geometry we consider a linearly expanding $(D+1)$-dimensional universe described by the line element

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2, a(t) = bt \ , \tag{1}$$

with spatial coordinates $\mathbf{x} = (x^1, \cdots, x^D)$. In (1), $0 \le t < \infty$ and $b > 0$ is a constant having dimension of inverse length. Introducing a conformal time η, $-\infty < \eta < +\infty$, in accordance with

$$t = e^{b\eta}/b \ , \tag{2}$$

the line element is written in explicitly conformally flat form

$$ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2), a(\eta) = e^{b\eta}. \tag{3}$$

In what follows we will work in the spacetime coordinate system $(\eta, \mathbf{x})$. In these coordinates, the Ricci scalar, R, and the nonzero components of the Ricci tensor, $R_{\mu\nu}$, are given by the expressions

$$R = D(D-1)b^2 e^{-2b\eta}, R_{00} = 0, R_{ik} = -(D-1)b^2\delta_{ik}, \tag{4}$$

with $i, k = 1,2, \dots, D$. From the Einstein equations for the corresponding energy density ε and the pressure p one has

$$\varepsilon = \frac{D(D-1)}{16\pi G t^2}, \ p = -\frac{D-2}{D}\varepsilon, \tag{5}$$

where G is the Newton gravitational constant. For $D = 1$, the geometry we have described is flat and coincides with the $(1+1)$-dimensional Milne universe. We will consider a massive scalar field $\phi(x)$ with the curvature coupling parameter ξ. The corresponding field equation reads

$$(\nabla_\mu \nabla^\mu + m^2 + \xi R)\phi = 0, \tag{6}$$

where ∇_μ stands for the covariant derivative operator. Here we are interested in the effects on the scalar vacuum induced by codimension one flat boundaries (plates) located at $x^D \equiv z = z_1$ and $z = z_2$ $(z_2 > z_1)$. On the plate $z = z_j, j = 1,2,$ the field operator is constrained by the boundary condition $(1 + \beta_j' n_j^\mu \nabla_\mu)\phi = 0$, with n_j^μ being the normal to the boundary obeying the relation $n_{j\mu} n_j^\mu = -1$. The boundary conditions considered are of the Robin type and generalize the Dirichlet $(\beta'_j = 0)$ and Neumann $(\beta'_j = \infty)$ boundary conditions. In the regions $z < z_1$ and $z > z_2$ for the normal one has $n_1^\mu = -\delta_D^\mu e^{-b\eta}$ and $n_1^\mu = \delta_D^\mu e^{-b\eta}$, respectively. For the region $z_1 \leqslant z \leqslant z_2$ the normal is given by $n_j^\mu = (-1)^{j-1}\delta_D^\mu / a(\eta)$ for $j = 1,2$. The coefficients β'_j have the dimension of length and in some problems characterize the penetration depth of the field. In what follows a special case will be considered with the Robin coefficients $\beta'_j = \beta_j e^{b\eta} = \beta_j bt$ where β_j, $j = 1,2$, are constants (the penetration length scales proportional to the scale factor). In this case, the boundary conditions in the region between the plates take the form

$$\left(1 + (-1)^{j-1}\beta_j \partial_z\right)\phi = 0\,, z = z_j. \tag{7}$$

Scalar modes. The background space is flat and for the complete set of scalar modes in the region between the plates, $z_1 \leqslant z \leqslant z_2$, the dependence on the spatial coordinates can be taken similar to that for plates in the Minkowski bulk:

$$\phi(x) = Cf(\eta)e^{i\mathbf{k}\cdot\mathbf{x}_{||}} \cos\left[\lambda\left(z - z_j\right) + \alpha_j(\lambda)\right], \tag{8}$$

where $\mathbf{k} = (k^1, k^2, \cdots, k^{D-1})$, $\mathbf{x}_{||} = (x^1, x^2, \cdots, x^{D-1})$, the function $\alpha_j(\lambda)$ is defined as

$$e^{2i\alpha_j(\lambda)} = \frac{i\lambda\beta_j(-1)^j + 1}{i\lambda\beta_j(-1)^j - 1}\,, \tag{9}$$

and C is the normalization constant. The modes (8) obey the boundary condition on the plate $z = z_j$. From the boundary condition on the second plate it follows that the eigenvalues of the quantum number λ are roots of the equation

$$(1 - b_1 b_2 u^2)\sin u - (b_1 + b_2)u\cos u = 0, \tag{10}$$

with the notations

$$u = \lambda z_0, b_j = \beta_j / z_0\,, z_0 = z_2 - z_1. \tag{11}$$

The equation (10) coincides with the eigenvalue equation for plates in the Minkowski bulk [2]. We will denote the roots of the transcendental equation (10) by $u = u_n$, $n = 1,2,\ldots$. For the eigenvalues of the quantum number λ one has $\lambda = \lambda_n = u_n / z_0$. In the discussion below we will assume the values of the parameters b_j for which all the roots u_n are real (for possible purely imaginary roots see [2]). In particular, this is the case for $\beta_j \leq 0$. In order to determine the function $f(\eta)$, we substitute (8) into the field equation (6). This leads to the equation

$$f''(\eta) + (D-1)bf'(\eta) + [\gamma^2 + a^2(m^2 + \xi R)]f(\eta) = 0\,, \tag{12}$$

where $\gamma = \sqrt{\lambda^2 + k^2}$, $k = |\mathbf{k}|$, and the prime stands for the derivative with respect to η. The solution of this equation is expressed in terms of cylindrical functions as

$$f(a) = (bt)^{(1-D)/2}\left[w_1 e^{-\nu\pi/2} H_{i\nu}^{(1)}(mt) + w_2 e^{\nu\pi/2} H_{i\nu}^{(2)}(mt)\right] , \quad (13)$$

with $H_{i\nu}^{(l)}(x)$, $l = 1,2$, being the Hankel functions, $\nu = \sqrt{\gamma^2 b^{-2} + \xi D(D-1) - (D-1)^2/4}$ and t is expressed in terms of the conformal time η as (2). The function $\nu = \nu(\eta)$ can be either positive or purely imaginary. In (13), the coefficients w_1 and w_2, in general, can be functions of γ. The factors $e^{\pm\nu\pi/2}$ are extracted for the further convenience. In what follows we will assume that the function $f(\eta)$ is normalized by the condition

$$f(\eta,\gamma)\partial_\eta f^*(\eta,\gamma) - f^*(\eta,\gamma)\partial_\eta f(\eta,\gamma) = ia^{1-D}, \quad (14)$$

where the star stands for the complex conjugate. Substituting (13) and using the Wronskian relation for the Hankel functions, one gets the relation between the coefficients

$$|w_2|^2 - |w_1|^2 = \frac{\pi}{4b} . \quad (15)$$

We can write the solution (13) in terms of the Bessel function $J_{i\nu}(z)$:

$$f(\eta,\gamma) = a^{(1-D)/2}[d_1 J_{-i\nu}(mt) + d_2 J_{i\nu}(mt)] , \quad (16)$$

where, again, t is given by (2). The coefficients d_1 and d_2 are related to the previous ones by the formulae

$$d_1 = \frac{w_2 e^{\nu\pi/2} - w_1 e^{-\nu\pi/2}}{\sinh(\nu\pi)} , d_2 = \frac{w_1 e^{\nu\pi/2} - w_2 e^{-\nu\pi/2}}{\sinh(\nu\pi)}, \quad (17)$$

$$w_1 = \frac{e^{-\nu\pi/2} d_1 + e^{\nu\pi/2} d_2}{2} , w_2 = \frac{e^{\nu\pi/2} d_1 + e^{-\nu\pi/2} d_2}{2}. \quad (18)$$

From (15) we obtain the following relation between the new coefficients

$$(|d_1|^2 - |d_2|^2)\sinh\left[(\nu+\nu^*)\frac{\pi}{2}\right] + (d_1 d_2^* - d_1^* d_2)\sinh\left[(\nu-\nu^*)\frac{\pi}{2}\right] = \frac{\pi}{2b}. \quad (19)$$

So, for the complete set of solutions one has $\left\{\phi_{n\mathbf{k}}^{(+)}(x), \phi_{n\mathbf{k}}^{(-)}(x) = \phi_{n\mathbf{k}}^{(+)*}(x)\right\}$ with

$$\phi_{n\mathbf{k}}^{(+)}(x) = Cf(\eta,\gamma)e^{i\mathbf{k}\cdot\mathbf{x}_{||}}\cos[\lambda_n(z-z_j) + \alpha_j(\lambda_n)], \quad (20)$$

where we have explicitly displayed the dependence of the function f on $\gamma_n = \sqrt{\lambda_n{}^2 + k^2}$. From the orthonormalization condition of the scalar modes, for the coefficient C one gets

$$|C|^2 = \frac{2}{(2\pi)^{D-1} z_0 c_n}, \; c_n = 1 + \frac{\sin u_n}{u_n}\cos[u_n + 2\tilde{\alpha}_j(u_n)] , \quad (21)$$

where the function $\tilde{\alpha}_j(u)$ is defined as $e^{2i\tilde{\alpha}_j(u_n)} = (iub_j - 1)/(iub_j + 1)$, with $j = 1,2$.

Asymptotics of the mode functions and the vacuum states

Adiabatic vacuum: First let us consider the Minkowskian limit. As seen from (3), in this limit $b \to 0$ for fixed η and, consequently, $mt \approx m/b + m\eta \gg 1$. For the function ν one has $\nu \approx \gamma/b$. This means that both the argument and the absolute value of the order for the Hankel functions in (13) are large. By using the uniform asymptotic expansions for the Hankel functions one gets

$$f(\eta,\gamma) \approx \sqrt{\frac{2b}{\pi\omega}}\left[w_1 e^{i\nu\xi(m/\gamma) - i\pi/4} e^{i\omega\eta} + w_2 e^{-i\nu\xi(m/\gamma) + i\pi/4} e^{-i\omega\eta}\right], \quad (22)$$

where $\omega = \sqrt{\gamma^2 + m^2}$ and $\xi(u) = \sqrt{1+u^2} + \ln\left(\frac{u}{1+\sqrt{1+u^2}}\right)$. For the coefficients we have the relation (15). From (22) it follows that the state under consideration is reduced to the

Minkowskian vacuum if $w_1 = 0$. The vacuum state obeying this property is called an adiabatic vacuum. For this vacuum $|w_2|^2 \approx \pi/(4b)$.

Conformal vacuum: Consider a conformally coupled massless field for which $\xi = (D-1)/(4D)$ and, hence, $\nu = \gamma/b$. By using the asymptotic expression for the Bessel function for small arguments [3], from (16) in the limit $m \to 0$ one gets

$$f(\eta,\gamma) = a^{(1-D)/2}\left[\frac{d_1 e^{-i\nu\ln(m/2b)}}{\Gamma(1-i\nu)}e^{-i\gamma\eta} + \frac{d_2 e^{i\nu\ln(m/2b)}}{\Gamma(1+i\nu)}e^{i\gamma\eta}\right], \quad (23)$$

where $\Gamma(x)$ is the gamma function. The corresponding modes are conformally related to the positive-energy mode functions in the Minkowski bulk if $d_2 = 0$. This correspond to the following relation for the coefficients $w_{1,2}$:

$$w_2 = w_1 e^{\nu\pi}. \quad (24)$$

From (19) one finds $|d_1|^2 = \frac{\pi}{2b\sinh(\gamma\pi/b)}$. By taking into account that $\Gamma(1+i\gamma/b)\Gamma(1-i\gamma/b) = \frac{\gamma\pi/b}{\sinh(\gamma\pi/b)}$, for the corresponding modes, up to a phase, one gets

$$f(\eta,\gamma) = a^{(1-D)/2}\frac{e^{-i\gamma\eta}}{\sqrt{2\gamma}}. \quad (25)$$

The vacuum state defined by the mode functions with $d_2 = 0$ is called a conformal vacuum. For the corresponding modes we get

$$f(\eta,\gamma) = f_C(\eta,\gamma) = d_1(bt)^{(1-D)/2}J_{-i\nu}(mt), \quad (26)$$

where $|d_1|^2\sinh\left[(\nu+\nu^*)\frac{\pi}{2}\right] = \frac{\pi}{2b}$. From here it follows that the conformal vacuum is physically realizable for real values of ν only and the mode functions are given by (26) with $|d_1|^2 = \frac{\pi}{2b\sinh(\nu\pi)}$.

Two-point function: As a two-point function we will consider the Hadamard function defined as the VEV $G(x,x') = \langle 0|\phi(x)\phi(x') + \phi(x')\phi(x)|0\rangle$. Expanding the field operator in terms of the complete set of mode functions and using the commutation relations for the annihilation and creation operators, the following mode-sum formula is obtained:

$$G(x,x') = \int d\mathbf{k}\sum_{n=1}^{\infty}\sum_{s=\pm}\phi_\sigma^{(s)}(x)\phi_\sigma^{(s)*}(x'), \quad (27)$$

with the mode functions given by (20). Substituting them one gets the representation

$$G(x,x') = \frac{(tt')^{(1-D)/2}}{(2\pi b)^{D-1}z_0}\int d\mathbf{k}\ e^{i\mathbf{k}\cdot\Delta\mathbf{x}_{||}}\sum_{n=1}^{\infty}\frac{w(t,t',\gamma_n)}{c_n}\{\cos(\lambda_n z) + \cos[\lambda_n(z+z'-2z_j) + 2\alpha_j(\lambda_n)]\},$$

(28)

where $\Delta\mathbf{x}_{||} = \mathbf{x}_{||} - \mathbf{x}'_{||}$, $\Delta z = z - z'$ and

$$W(t,t',\gamma) = [|w_1|^2 + |w_2|^2][H_{i\nu}^{(1)}(mt)H_{i\nu}^{(2)}(mt') + H_{i\nu}^{(1)}(mt')H_{i\nu}^{(2)}(mt)] + 2w_1w_2^*e^{-\nu\pi}H_{i\nu}^{(1)}(mt)H_{i\nu}^{(1)}(mt') + 2w_1^*w_2e^{\nu\pi}H_{i\nu}^{(2)}(mt)H_{i\nu}^{(2)}(mt'). \quad (29)$$

For the adiabatic vacuum $w_1 = 0$ and we find $W(t,t',\gamma) = \frac{\pi}{4b}[H_{i\nu}^{(1)}(mt)H_{i\nu}^{(2)}(mt') + H_{i\nu}^{(1)}(mt')H_{i\nu}^{(2)}(mt)]$. For the conformal vacuum

$$w_1 = e^{-\nu\pi/2}\, d_1/2 \ , w_2 = e^{\nu\pi/2}\, d_1/2 \ , \tag{30}$$

and the function $W(t,t',\gamma)$ is given by

$$W(t,t',\gamma) = \pi \frac{J_{-i\nu}(mt)J_{i\nu}(mt')+J_{i\nu}(mt)J_{-i\nu}(mt')}{2b\sinh(\nu\pi)} \ . \tag{31}$$

Recall that for the conformal vacuum ν should be real. In the further discussion we will consider the conformal vacuum and a conformally coupled scalar field. For the latter $\xi = (D-1)/(4D)$ and $\nu = \gamma/b$. Hence, the function in the expression (28) of the Hadamard function takes the form

$$W(t,t',\gamma) = \pi \frac{J_{-i\gamma/b}(mt)J_{i\gamma/b}(mt')+J_{i\gamma/b}(mt)J_{-i\gamma/b}(mt')}{2b\sinh(\nu\pi)} \ . \tag{32}$$

The Hadamard function (28) with (32) is further transformed by using a variant of the generalized Abel-Plana summation formula [2, 4]:

$$\sum_{n=1}^{\infty} \frac{g(u_n)}{c_n} = -\frac{g(0)/2}{1-b_2-b_1} + \frac{1}{\pi}\int_0^{\infty} du\, g(u) + \frac{i}{\pi}\int_0^{\infty} du \frac{g(iu)-g(-iu)}{c_1(u)c_2(u)e^{2u}-1} \ , \tag{33}$$

with the notation $c_j(u) = \frac{b_j u-1}{b_j u+1}$. In this formula we take the function

$$g(u) = \left\{\cos(u\Delta z/z_0) + \cos\left[u\left(z+z'-2z_j\right)/z_0 + 2\alpha_j(u/z_0)\right]\right\} W\left(t,t',\sqrt{u^2/z_0{}^2+k^2}\right),$$

(34)

For the latter one has $g(iu) - g(-iu) = 0$ for $u < kz_0$ and $2g(iu)$ for $u > kz_0$. In deriving the summation formula (33) from the generalized Abel-Plana formula in [2, 4], it was assumed that the function $g(u)$ is analytic in the right half of the complex plane $Re u \geq 0$ and obeys the condition $|g(u)| < \epsilon(x)e^{c|y|}$ for $|u| \to \infty$, where $u = x+iy$, $c > 2$ and $\epsilon(x) \to 0$ for $x \to \infty$. The function (34) obeys these conditions except the analyticity on the imaginary axis $Re u = 0$: the function $g(u)$ has simple poles $u = \pm iy_l$, $l = 1,2,\cdots$, with $y_l = z_0\sqrt{k^2 + l^2b^2}$, coming from the zeros of the denominator in (32). In the derivation of the summation formula (33) from the generalized Abel-Plana formula (see [4]) the poles $\pm iy_l$ should be excluded by small semicircles $C_\rho^{\pm}$ with radius ρ on the right half-plane, with the subsequent limiting transition $\rho \to 0$. The contributions of the integrals along these semicircles to the right-hand side of (33) is expressed as

$$\frac{1}{2}\sum_{j=+,-}\int_{C_\rho^j} du \frac{h_j(u)}{\sinh\left(\pi\sqrt{u^2/z_0{}^2+k^2}/b\right)}, \tag{35}$$

where

$$h_{\pm}(u) = \frac{\pi i}{2b}(b_1 u \pm i)(b_2 u \pm i)e^{\pm iu} \times \frac{J_{-i\gamma/b}(mt)J_{i\gamma/b}(mt')+J_{i\gamma/b}(mt)J_{-i\gamma/b}(mt')}{(1-b_1b_2u^2)\sin u-(b_1+b_2)u\cos u} \ . \tag{36}$$

For the separate integrals one has

$$\int_{C_\rho^{\pm}} du \frac{h_{\pm}(u)}{\sinh(\pi\gamma/b)} = ilb^2 z_0{}^2 \frac{h_{\pm}(\pm iy_l)}{(-1)^l y_l} \ . \tag{37}$$

Now, it can be seen that $h_-(-iw_l) = -h_+(iw_l)$ and, hence, in (5.12) the contributions coming from the poles iy_l and $-iy_l$ cancel each other. From here we conclude that the summation formula (33) is valid for the function (34) if the last integral in the right-hand side is understood in the sense of the principal value. Applying the summation formula (33) to the series in (28) and introducing the function

$$V(t,t',\chi)=\frac{J_{\chi}(mt)J_{-\chi}(mt')+J_{-\chi}(mt)J_{\chi}(mt')}{\sin(\pi\chi)}\ , \tag{38}$$

the Hadamard function is presented in the form

$$G(x,x')=G_j(x,x')+\frac{(b^2tt')^{(1-D)/2}}{(2\pi)^{D-1}bz_0}\int d\mathbf{k}\int_{kz_0}^{\infty}du\frac{V(t,t',\chi)e^{i\mathbf{k}\cdot\Delta\mathbf{x}_{||}}}{c_1(u)c_2(u)e^{2u}-1}\left[\cosh(u\Delta z/z_0)+\sum_{s=\pm1}c_j^s(u)e^{su|z+z'-2z_j|/z_0}\right]\ . \tag{39}$$

In the integrand we have defined $\chi=b^{-1}\sqrt{u^2/z_0{}^2-k^2}$. The first term in the right-hand side comes from the first integral in (33). It is further decomposed as

$$G_j(x,x')=G_0(x,x')+\frac{(tt')^{(1-D)/2}}{(2\pi)^Db^{D-1}}\int d\mathbf{k}\,e^{i\mathbf{k}\cdot\Delta\mathbf{x}_{||}}\int_0^{\infty}dy\sum_{s=\pm1}e^{siy(z+z'-2z_j)}\frac{iy\beta_j+s(-1)^j}{iy\beta_j-s(-1)^j}W\left(t,t',\sqrt{y^2+k^2}\right)\ , \tag{40}$$

where

$$G_0(x,x')=\frac{(b^2tt')^{(1-D)/2}}{(2\pi)^{D/2}|\Delta\mathbf{x}|^{D/2-1}}\int_0^{\infty}du\,u^{D/2}J_{D/2-1}(u|\Delta\mathbf{x}|)W(t,t',u)\ , \tag{41}$$

with $\Delta\mathbf{x}=(\Delta\mathbf{x}_{||},x^D-x'^D)$, is the Hadamard function in the geometry (3) without boundaries. In the limit $z_0\to\infty$, the second term in the right-hand side of (39) vanishes whereas the term $G_j(x,x')$ depends on the location z_j of a single plate only. From here it follows that the function $G_j(x,x')$ corresponds to the Hadamard function for the geometry of a single plate at $z=z_j$. It can be presented in an alternative form rotating the integration contour by the angle $\pi/2$ for the term with $s=+1$ and by the angle $-\pi/2$ for the term with $s=-1$. The poles $y=\pm i\sqrt{k^2+l^2b^2}$ on the imaginary axis are excluded by small semicircles in the right-half plane. In a way similar to that we have used above, it can be seen that the contributions from the poles with the upper and lower signs cancel each other and one gets the representation

$$G_j(x,x')=G_0(x,x')+\frac{(b^2tt')^{(1-D)/2}}{2(2\pi)^{D-1}b}\int d\mathbf{k}\,e^{i\mathbf{k}\cdot\Delta\mathbf{x}_{||}}\int_k^{\infty}dy\frac{\beta_jy+1}{\beta_jy-1}e^{-y|z+z'-2z_j|}V\left(t,t',b^{-1}\sqrt{y^2-k^2}\right)\ , \tag{42}$$

where the integral over y is understood in the sense of the principal value. Substituting this representation into (39), the Hadamard function in the region between two plates is presented in the form

$$G(x,x')=G_0(x,x')+\frac{(b^2tt')^{(1-D)/2}}{(2\pi)^{D-1}bz_0}\int d\mathbf{k}\int_{kz_0}^{\infty}du\frac{V(t,t',\chi)e^{i\mathbf{k}\cdot\Delta\mathbf{x}_{||}}}{c_1(u)c_2(u)e^{2u}-1}\left[\cosh(u\Delta z/z_0)+\frac{1}{2}\sum_{j=1,2}c_j(u)e^{u|z+z'-2z_j|/z_0}\right] \tag{43}$$

In the regions $z<z_1$ and $z>z_2$, the Hadamard function is given by (42) with $j=1$ and $j=2$, respectively. The renormalization is required for the boundary-free contributions only.

VEV of the field squared: In this and following sections we will investigate the local characteristics of the vacuum state. As such, first we consider the VEV of the field squared, denoted here as $\langle 0|\phi^2|0\rangle \equiv \langle \phi^2 \rangle$ (in what follows the index C in the notation of the conformal vacuum state will be omitted). In the region between the plates, taking the coincidence limit $x' \to x$ in the arguments of the Hadamard function (43), one gets

$$\langle \phi^2 \rangle = \langle \phi^2 \rangle_0 + \langle \phi^2 \rangle_b \ , \tag{44}$$

where $\langle \phi^2 \rangle_0$ is the renormalized VEV in the absence of the boundaries and the boundary-induced contribution is given by the expression

$$\langle \phi^2 \rangle_b = \frac{B_D t^{D-1}}{(z_0 b)^D} \int_0^\infty dx\, x^{D-2} \int_x^\infty du \frac{U\left(mt, \sqrt{u^2-x^2}/(bz_0)\right)}{c_1(u)c_2(u)e^{2u}-1} c(u,z),$$

$$B_D = \frac{(4\pi)^{(1-D)/2}}{\Gamma((D-1)/2)}, U(x,y) = \frac{J_y(x)J_{-y}(x)}{\sin(\pi y)}. \tag{45}$$

Note that the background geometry is homogeneous and the boundary-free part $\langle \phi^2 \rangle_0$ does not depend on the spatial point. The boundary-induced contribution in (45) is further transformed passing to a new integration variable $y = \sqrt{u^2 - x^2}$ and introducing polar coordinates in the plane (x, y). This leads to the result

$$\langle \phi^2 \rangle_b = \frac{B_D t^{D-1}}{(bz_0)^D} \int_0^\infty du \frac{u^{D-1}\, S_D(mt, u/(bz_0)) c(u,z)}{c_1(u)c_2(u)e^{2u}-1} \ , \tag{46}$$

with the notation

$$S_D(mt, x) = \int_0^1 ds\, s(1-s^2)^{(D-3)/2} U(mt, xs). \tag{47}$$

For a massless field

$$S_D(mt, x) = \frac{\Gamma((D-1)/2)}{2\sqrt{\pi}\Gamma(D/2)x} \ , \tag{48}$$

and we can see that the boundary-induced term in (46) is connected to the corresponding result in the Minkowski bulk, $\langle \phi^2 \rangle_b$, by the conformal relation $\langle \phi^2 \rangle_b = (bt)^{1-D} \langle \phi^2 \rangle_b^{(M)}$, where

$$\langle \phi^2 \rangle_b^{(M)} = \frac{(4\pi)^{-D/2}}{\Gamma(D/2) z_0^{D-1}} \int_0^\infty du \frac{u^{D-1} c(u,z)}{c_1(u)c_2(u)e^{2u}-1} \ . \tag{49}$$

In the regions $z < z_1$ and $z > z_2$, the VEV of the field squared is obtained from (42). For these regions we have the decomposition

$$\langle \phi^2 \rangle_j = \langle \phi^2 \rangle_0 + \langle \phi^2 \rangle_{bj}, \tag{50}$$

with the boundary-induced part

$$\langle \phi^2 \rangle_{bj} = \frac{B_D}{b^D t^{D-1}} \int_0^\infty dk\, k^{D-2} \int_k^\infty dy \frac{\beta_j y+1}{\beta_j y-1} e^{-2y|z-z_j|} U\left(mt, \sqrt{y^2-k^2}/b\right) \ . \tag{51}$$

Here $j = 1$ for the region $z < z_1$ and $j = 2$ for the region $z > z_2$. With a transformation similar to that used for (46), the expression (51) can also be presented as

$$\langle \phi^2 \rangle_{bj} = \frac{B_D}{b^D t^{D-1}} \int_0^\infty dy\, y^{D-1} S_D(mt, y/b) \frac{\beta_j y+1}{\beta_j y-1} e^{-2y|z-z_j|} \ . \tag{52}$$

For a massless field, by using (48), we obtain the standard relation with the corresponding result in Minkowski spacetime, $\langle\phi^2\rangle_{bj} = (bt)^{1-D}\langle\phi^2\rangle_{bj}^{(M)}$, where

$$\langle\phi^2\rangle_{bj}^{(M)} = \frac{(4\pi)^{-D/2}}{\Gamma(D/2)}\int_0^\infty dy\, y^{D-2}\frac{\beta_j y+1}{\beta_j y-1}e^{-2y|z-z_j|}\ , \tag{53}$$

is the Minkowskian VEV for a massless field.

VEV of the energy-momentum tensor: The VEV of the energy-momentum tensor is expressed in terms of the Hadamard function and the VEV of the field squared as

$$\langle T_{\mu\nu}\rangle = \frac{1}{2}\lim_{x'\to x}\partial_\mu\partial'_\nu G(x,x') - \frac{1}{4D}\left[g_{\mu\nu}\nabla_l\nabla^l - (D-1)\left(\nabla_\mu\nabla_\nu + R_{\mu\nu}\right)\right]\langle\phi^2\rangle, \tag{54}$$

where the components of the Ricci tensor are given by (4). We first consider the VEV in the regions $z<z_1$and $z>z_2$. By using the expression (42) for the corresponding Hadamard function, the vacuum energy-momentum tensor is decomposed as $\langle T_\mu^\nu\rangle = \langle T_\mu^\nu\rangle_0 + \langle T_\mu^\nu\rangle_{bj}$, where $\langle T_\mu^\nu\rangle_0$ is the VEV in the absence of boundaries and $\langle T_\mu^\nu\rangle_{bj}$ is induced by the plate at $z=z_j$, $j=1,2$. For the diagonal components of the boundary-induced contribution one gets (no summation over μ)

$$\langle T_\mu^\mu\rangle_{bj} = \frac{B_D}{b^D t^{D-1}}\int_0^\infty dk\, k^{D-2}\int_k^\infty dy\frac{\beta_j y+1}{\beta_j y-1}e^{-2y|z-z_j|}\left(\hat{f}_\mu + \frac{h_\mu y^2 + c_\mu k^2}{b^2}\right)U\left(mt,\sqrt{y^2-k^2}/b\right)\ , \tag{55}$$

where $h_0 = -\frac{D-1}{D}, h_l = \frac{1}{D}, h_D = 0, c_0 = 1,\ c_l = \frac{1}{1-D}, c_D = 0,\ l = 1,\cdots,D-1$. The operators in (55) are defined by the expressions $\hat{f}_0 = \frac{1}{4}(t^2\partial_t^2 + t\partial_t) + t^2m^2,\ \hat{f}_\mu = -\frac{1}{4D}(t^2\partial_t^2 + t\partial_t), \mu\neq 0$. Due to the homogeneity of the background spacetime, the boundary-free contribution $\langle T_\mu^\nu\rangle_0$ does not depend on the spatial point and the spatial components are isotropic. The problem under consideration is inhomogeneous along the t - and z -directions. As a consequence of that, in addition to the diagonal components, the vacuum energy-momentum tensor has a nonzero off-diagonal component

$$\langle T_0^D\rangle_{bj} = \frac{sgn(z-z_j)B_D}{2D(bt)^{D+1}}\int_0^\infty dk\, k^{D-2}\int_k^\infty dy\, y\frac{\beta_j y+1}{\beta_j y-1}e^{-2y|z-z_j|}t\partial_t U\left(mt,\sqrt{y^2-k^2}/b\right). \tag{56}$$

The off-diagonal component (56) corresponds to the energy flux along the direction perpendicular to the plate. It has different signs in the regions $z<z_j$ and $z>z_j$. The energy flux can be either directed from the plate or to the plate. If $\langle T_0^D\rangle_{bj}>0$ ($\langle T_0^D\rangle_{bj}<0$) in the region $z>z_j$, the energy flux is directed from (to) the plate in both the regions $z<z_j$ and $z>z_j$. Alternative expressions for the VEVs in the regions $z<z_1$ and $z>z_2$, are

$$\langle T_\mu^\mu\rangle_{bj} = \frac{B_D}{b^{D+2}t^{D+1}}\int_0^\infty dy\, y^{D+1}\frac{\beta_j y+1}{\beta_j y-1}e^{-2y|z-z_j|}\left\{c_\mu S_{D+2}(mt,\ y/b) + \left[(b/y)^2\hat{f}_\mu + h_\mu\right]S_D(mt,\ y/b)\right\},$$
$$\langle T_0^D\rangle_{bj} = \frac{sgn(z-z_j)B_D}{2D(bt)^{D+1}}\int_0^\infty dy\, y^D\frac{\beta_j y+1}{\beta_j y-1}e^{-2y|z-z_j|}t\partial_t S_D(mt,\ y/b). \tag{57}$$

Now let us consider the region between the plates, $z_1\le z\le z_2$. By taking into account the expression (43) for the Hadamard function and using (54), the vacuum energy-momentum tensor is presented as

$$\langle T_\mu^\nu\rangle = \langle T_\mu^\nu\rangle_0 + \langle T_\mu^\nu\rangle_b\ . \tag{58}$$

The diagonal components of the boundary-induced contribution are given by the formula (no summation over μ)

$$\langle T_\mu^\mu\rangle_b = \frac{B_D}{(bz_0)^D t^{D+1}}\int_0^\infty dx\, x^{D-2}\int_x^\infty du\, \left\{c(u,z)\left[\hat{f}_\mu + \frac{h_\mu u^2 + c_\mu x^2}{(bz_0)^2}\right] - \frac{2d_\mu u^2}{(bz_0)^2}\right\}\frac{U\left(mt, \sqrt{u^2-x^2}/(bz_0)\right)}{c_1(u)c_2(u)e^{2u}-1}, \quad (59)$$

with $d_\mu = 1/D$ for $\mu \neq D$ and $d_D = -1$. In addition, where is a nonzero off-diagonal component corresponding to energy flux perpendicular to the plates:

$$\langle T_0^D\rangle_b = -\frac{B_D}{2D(btz_0)^{D+1}}\int_0^\infty dx\, x^{D-2}\int_x^\infty du\, u\frac{\sum_{j=1,2} sgn(z-z_j)c_j(u)e^{2u|z-z_j|/z_0}}{c_1(u)c_2(u)e^{2u}-1} t\partial_t U\left(mt, \sqrt{u^2-x^2}/(bz_0)\right) \quad .$$

(60)

Introducing in (59) and (60) a new integration variable $y = \sqrt{u^2 - x^2}$ and passing to polar coordinates in the (x, y)-plane, we obtain equivalent representations (no summation over μ)

$$\langle T_\mu^\mu\rangle_b = \frac{B_D}{(z_0 b)^{D+2}t^{D+1}}\int_0^\infty du\, \frac{u^{D+1}}{c_1(u)c_2(u)e^{2u}-1}$$
$$\times\left\{c_\mu c(u,z)S_{D+2}(mt,\, u/(bz_0)) + \left[c(u,z)\left((b\, z_0/u)^2\hat{f}_\mu + h_\mu\right) - 2d_\mu\right]S_D(mt,\, u/(bz_0))\right\}, \quad (61)$$

$$\langle T_0^D\rangle_b = -\frac{B_D}{2D(btz_0)^{D+1}}\int_0^\infty du\, u^D\frac{\sum_{j=1,2} sgn(z-z_j)c_j(u)e^{2u|z-z_j|/z_0}}{c_1(u)c_2(u)e^{2u}-1} t\partial_t S_D(mt,\, u/(bz_0)). \quad (62)$$

For a massive field the VEVs (61) and (62) diverge on the plates. The divergences on the plate at $z = z_j$ are the same as those for $\langle T_\mu^\nu\rangle_{bj}$. By using the relations $\sum_{\mu=0}^D \hat{f}_\mu = t^2 m^2$ and $\sum_{\mu=0}^D h_\mu = \sum_{\mu=0}^D d_\mu = 0$, we can check that the boundary-induced contributions in all the regions obey the trace relation

$$\langle T_\mu^\mu\rangle_b = m^2\langle\phi^2\rangle_b. \quad (63)$$

The boundary-induced VEVs satisfy the covariant conservation equation $\nabla_\mu\langle T_\nu^\mu\rangle_b = 0$, which is reduced to the following two equations

$$t^{-D}\partial_t(t^{D+1}\langle T_0^0\rangle_b) + \frac{1}{b}\partial_z\langle T_0^D\rangle_b - \langle T_\mu^\mu\rangle_b = 0,\ t^{-D}\partial_t(t^{D+1}\langle T_0^D\rangle_b) - \frac{1}{b}\partial_z\langle T_D^D\rangle_b = 0. \quad (64)$$

The second of these equations shows that the inhomogeneity of the normal stress is related to the nonzero energy flux along the direction normal to the plates.

The Casimir forces: The vacuum force acting per unit surface of the plate at $z = z_j$ is determined by the normal stress $\langle T_D^D\rangle|_{z=z_j}$. For a massive field this quantity diverges. The divergence comes from the single plate contribution $\langle T_D^D\rangle_{bj}$. The latter is the same on the left- and right-hand sides of the plate and, hence, the corresponding net force is zero. The same is the case for the boundary free part $\langle T_D^D\rangle_0$. Consequently, the resulting force comes from the second plate-induced part $\langle T_D^D\rangle - \langle T_D^D\rangle_j$ and the corresponding effective pressure is given by $(\langle T_D^D\rangle_j - \langle T_D^D\rangle)|_{z=z_j}$, where $\langle T_D^D\rangle$ is the normal stress in the region between the plates. The forces corresponding to P_j act on the sides of $z = z_1 + 0$ and $z = z_2 - 0$ of the plates. They are attractive (repulsive) for negative (positive) P_j. By taking into account the expressions (55) and (59), the vacuum pressures on the plates are presented as

$$P_j = -B_D\frac{(bz_0)^{-D}}{t^{D+1}}\int_0^\infty dx\, x^{D-2}\int_x^\infty du\, \frac{2(u/bz_0)^2 + [2 + c_j(u) + 1/c_j(u)]\hat{f}_D}{c_1(u)c_2(u)e^{2u}-1}U\left(mt, \sqrt{u^2-x^2}/(bz_0)\right). \quad (65)$$

An alternative expressions for the forces acting on the plates are obtained by using the normal stresses from (57) and (61):

$$P_j = -\frac{B_D}{(bz_0)^D t^{D+1}} \int_0^\infty du\ u^{D-1} \frac{2(u/bz_0)^2 + [2 + c_j(u) + 1/c_j(u)]\hat{f}_D}{c_1(u)c_2(u)e^{2u} - 1} S_D(mt,\ u/(bz_0)). \qquad (66)$$

In particular, one can have the situation when the forces are repulsive at small separations between the plates and attractive at large separations. For a massless field, by using the expression (48) for the function $S_D(mt, x)$, one gets $P_j = P_j^{(M)}/(bt)^{D+1}$, where

$$P_j^{(M)} = -\frac{2(4\pi)^{-D/2}}{\Gamma(D/2) z_0^{D+1}} \int_0^\infty du\ \frac{u^D}{c_1(u)c_2(u)e^{2u} - 1}, \qquad (67)$$

is the corresponding pressure for plates in the Minkowski bulk with the separation z_0. Note that in the problem under consideration $z_0 bt$ is the proper distance between the plates for a fixed t. For the Minkowski bulk the Casimir forces are the same for separate plates, independently on the values of the Robin coefficient. As seen from (67), in general, this is not the case for an expanding universe. At small separations between the plates, compared with the curvature radius of the background spacetime, one has $bz_0 \ll 1$. By taking into account that for $u \gg 1$ one has $J_u(z)J_{-u}(z)/sin(\pi u) \sim 1/(\pi u)$, for large x we get the asymptotic expression $S_D(mt, x) \approx \frac{\Gamma((D-1)/2)}{2\sqrt{\pi}\Gamma(D/2)x}$. By using this, to the leading order one gets $P_j \approx P_j^{(M)}/(bt)^{D+1}$, where $P_j^{(M)}$ is given by (67). In the limit under consideration the effects of gravity on the Casimir forces are small and the leading term coincides with that in the Minkowski bulk multiplied by the conformal factor. If in addition $z_0 \ll |\beta_j|$, the leading term is further simplified as

$$P_j \approx -\frac{D\zeta(D+1)\Gamma((D+1)/2)}{(4\pi)^{(D+1)/2}(z_0 bt)^{D+1}}, \qquad (68)$$

with $\zeta(x)$ being the Riemann zeta function. The same leading term is obtained for Dirichlet boundary conditions ($\beta_j = 0$). The corresponding forces are attractive. For the Dirichlet boundary condition on one plate and for non-Dirichlet boundary condition on the other the forces are repulsive at small separations.

Now let us consider the asymptotics of the forces for the separation between the plates larger than the curvature radius of the background geometry, $bz_0 \gg 1$. By taking into account that for $u \ll 1$ we have $J_u(z)J_{-u}(z)/sin(\pi u) \approx J_0^2(z)/(\pi u)$, for small values of x one obtains $S_D(mt, x) \approx \frac{\Gamma((D-1)/2)J_0^2(mt)}{2\sqrt{\pi}\Gamma(D/2)x}$. By using this in the integrand of (66), to the leading order, for non-Dirichlet boundary conditions ($\beta_j \neq 0$) one gets

$$P_j \approx -\frac{m^2[J_1^2(mt) - J_0^2(mt)]}{2D(4\pi)^{D/2}\Gamma(D/2)(z_0 bt)^{D-1}} \int_0^\infty du\ u^{D-2} \frac{2 + c_j(u) + 1/c_j(u)}{c_1(u)c_2(u)e^{2u} - 1}. \qquad (69)$$

In particular, for the Neumann boundary condition we find

$$P_j \approx \frac{2m^2\Gamma((D-1)/2)\zeta(D-1)}{D(4\pi)^{(D+1)/2}(z_0 bt)^{D-1}} [J_1^2(mt) - J_0^2(mt)]. \qquad (70)$$

The corresponding Casimir forces can be either attractive or repulsive. For a massless field the leading terms (69) and (70) vanish. For the Dirichlet boundary condition on both the plates one has $c_j(u) = -1$ and the leading term is given by

$$P_j \approx \frac{D\Gamma((D+1)/2)\zeta(D+1)}{(4\pi)^{(D+1)/2}(z_0 bt)^{D+1}} J_0^2(mt). \quad (71)$$

In this case the forces are attractive. Note that for plates in the Minkowski bulk the Casimir forces are attractive for both the Dirichlet and Neumann boundary conditions at all separations between the plates.

Conclusion: We have investigated combined effects of the background gravitational field and boundaries on the quantum properties of the scalar vacuum. As a background geometry a linearly expanding spatially flat universe is taken. The boundary geometry is given by two parallel plates on which the field obeys the Robin boundary conditions with the coefficients being linear functions of the proper time coordinate t. The evaluation of the VEVs is presented for the example of a conformally coupled scalar field in the conformal vacuum state. With the explicitly extracted boundary-free part in the Hadamard function, for points away from the boundaries, the renormalization of the local VEVs in the coincidence limit is reduced to the renormalization in the boundary-free geometry. We have considered the VEV of the field squared, the VEV of the energy-momentum tensor. We have explicitly shown that the boundary-induced contributions obey the trace relation (63) and the covariant conservation equation. We have also investigated the Casimir forces. Unlike to the problem in the Minkowski bulk, for a massive field the Casimir force acting on the left and right plates are different if the Robin coefficients differ.

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BOUNDARY INDUCED QUANTUM EFFECTS IN LINEARLY EXPANDING COSMOLOGICAL MODELS

Key words: Casimir effect, Friedmann-Robertson-Walker spacetime, scalar field.

We investigate quantum vacuum effects for a massive scalar field, induced by two planar boundaries in a linearly expanding spatially flat spacetime. For the Robin boundary conditions, the mode functions and Hadamard function are presented. The conformal vacuum states is considered. The energy-momentum tensor has off-diagonal component describing energy flux along the direction perpendicular to the plates. The Casimir forces are different if the Robin coefficients are different. During the cosmological expansion, the forces may change the sign.

Պետրոսյան Տիգրան

ՍԱՀՄԱՆՆԵՐՈՎ ՄԱԿԱԾՎԱԾ ՔՎԱՆՏԱՅԻՆ ԵՐԵՎՈՒՅԹՆԵՐ ԳԾԱՅԻՆ ԸՆԴԱՐՁԱԿՎՈՂ ԿՈՍՄՈԼՈԳԻԱԿԱՆ ՄՈԴԵԼՆԵՐՈՒՄ

Բանալի բառեր՝ Կազիմիրի էֆեկտ, Ֆրիդման-Ռոբերտսոն-Ուոքերի տարածաժամանակ, սկալյար դաշտ:

Հետազոտված են զանգվածեղ սկալյար դաշտի համար քվանտային վակուումի երևույթներ գծայնորեն ընդարձակվող և տարածականորեն հարթ տարածաժամանակի ֆոնի վրա՝ մակածված երկու հարթ սահմաններով: Ներկայացված են մոդային ֆունկցիաները և Հադամարի ֆունկցիան Ռոբինի սահմանային պայմանների համար: Քննարկվում է կոնֆորմ վակուումային վիճակը: Էներգիա-իմպուլսի թենզորն ունի ոչ-զրոյական ոչ-անկյունագծային բաղադրիչ, որը նկարագրում է թիթեղների նորմալի ուղղությամբ էներգիայի հոսքը: Առանձին թիթեղների վրա ազդող Կազիմիրի ուժերը չեն համընկնում, երբ համապատասխան Ռոբինի գործակիցները տարբեր են: Կոսմոլոգիական ընդարձակման ընթացքում ուժերը կարող են փոխել նշանները:

Петросян Тигран

ИНДУЦИРОВАННЫЕ ГРАНИЦАМИ КВАНТОВЫЕ ЭФФЕКТЫ В ЛИНЕЙНО РАСШИРЯЮЩИХСЯ КОСМОЛОГИЧЕСКИХ МОДЕЛЯХ

Ключевые слова: эффект Казимира, пространство-время Фридмана-Робертсона-Уокера, скалярное поле.

Исследованы эффекты квантового вакуума для массивного скалярного поля на фоне линейно расширяющегося и пространственно плоского пространства-времени, индуцированные двумя плоскими границами. Приведены модовые функции и функция Адамара для граничных условий Робина. Рассмотрено конформное вакуумное состояние. Тензор энергии-импульса имеет ненулевую не диагональную компоненту, описывающую поток энергии по нормали к пластинам. Силы Казимира, действующие на отдельные пластины, не совпадают, когда соответствующие коэффициенты Робина разные. В ходе космологического расширения знаки сил могут меняться.