▸ ANAHIT CHUBARYAN, GARIK PETROSYAN, *On proof complexities for some classes of tautologies in Frege systems.*
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One of the most fundamental problems of the proof complexity theory is to find for classical propositional calculus a proof system, which has a polynomial size $p(n)$ proof for every tautology of size n. Cook and Reckhow named such a system a *super system* and showed in [1] that $NP = coNP$ iff a super system exists. Lately it is proved in [2] that $NP = coNP = PSPACE$, hence a super system must be. It is well known that many systems are not super. This question about Frege system, the most natural calculi for propositional logic, is still open.

Some results about Frege proof complexities are presented here. We introduce the notion of *specific* tautologies A in the form: $A = p \supset (A_1 \vee A_2 \vee \cdots \vee A_k)$ $(k \geqslant 1)$, where p is a literal (variable or negation of variable), neither $A_1 \vee A_2 \vee \cdots \vee A_k$ nor every $A_i(1 \leqslant i \leqslant k)$ are tautology or contradiction and $|A_i| \leqslant \frac{|A_1|}{2^{i-1}}$, and show that Frege systems are super systems iff there is a polynomial $p()$ such that all specific tautologies of size n have a proofs, size of which are bounded by $p(n)$. Then we show, that all *balanced* tautologies (every variable of which has only one positive and one negative occurrences), given in disjunctive normal form, also have Frege proofs with polynomial bounded sizes. Lastly we give some notes about relations between the proof complexities of tautologies A_n and B_n and proof complexities of the tautologies in a form $A_n * B_n$, where $*$ is $\wedge$, $\vee$, $\supset$. In particular we show that for some tautologies A_n and B_n proofs of formulas $A_n \vee B_n$ can be more easier than proofs every of A_n and B_n.

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[1] S.A. Cook, A.R. Reckhow, *The relative efficiency of propositional proof systems*, ***Journal of Symbolic logic***, Vol. 44, 1979, pp. 36-50.

[2] L. Gordeev, E. Haeusler, *NP vs PSPACE*, ***arXiv:1609.09562v1 [cs.CC]***, 30 Sep 2016.