

Electron Mobility Non-Damping Fluctuations in Semiconductors

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Equilibrium state disturbance and restoration processes of an electron gas interacting with phonon system in an equilibrium semiconductor have been investigated. Damping peculiarities of small deviations (fluctuations) of electron system from equilibrium state due to electron-acoustic phonon random scattering have been analyzed. A second-order linear partial differential equation for symmetric component of fluctuation of electron distribution function was obtained via linearization of the Boltzmann equation. This equation describes the chaotic movement of electrons along the energy axis, which can be interpreted as diffusion in momentum space. Another equation which describes the time dependence of electron lattice mobility fluctuations was obtained. It was shown that in the Boltzmann equation linearization approximation, lattice mobility fluctuations do not decay over time. The time dependence of the mobility fluctuations is described by stochastic harmonic function with random amplitude and random initial phase.

Keywords: Semiconductor; electron mobility fluctuation; electron–phonon scattering.

1. Introduction

The results of various recently published experimental studies (see, e.g., Refs. 1, 2) argue that the development of the theory of electron mobility fluctuations could serve as good basis for further explanation of the basic mechanism of low-frequency current noises in semiconductors. The main sources of carrier mobility fluctuations are: Random intraband scattering, generation–recombination transitions and the shot effect [3, 4], out of which electron–phonon intraband scattering has significant and fundamental importance. It is discovered [5, 6] that unlike electron concentration fluctuations, lattice mobility fluctuations attenuate over time more slowly,

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by non-exponential law. First of all, in order to specify the nature of this peculiarity that has a very important meaning for the theory of low-frequency $1/f$ noises [7], it is necessary to clarify the role of the phonon subsystem in the interacting electron–phonon system of the semiconductor. For that purpose electron lattice mobility fluctuations, conditioned by electron–phonon random scattering, are considered in this paper. As a result of phonon absorption or emission by electron the equilibrium of both electron and phonon systems is disturbed. On the other hand, as it is known [8], as a result of phonon–phonon intensive interaction the τ_{ph} phonon relaxation time, as a rule, is smaller than electron relaxation time τ , $\tau_{\text{ph}} \ll \tau$. It means that the disturbance of the phonon subsystem equilibrium restores much faster than the disturbance of the equilibrium electron system. Therefore, here neglecting the phonon fluctuations it has been assumed that the phonon system is in equilibrium state.

2. Electron Mobility Fluctuations

It was established [9] that in general cases electron mobility fluctuation $\tilde{\mu}$ is the result of the fluctuations of the electron quasi-momentum relaxation time τ_k ($\tilde{\tau}_k$ -source) and the occupancy of energy levels of the conduction band n_k (\tilde{n}_k -source). As shown in Ref. 9 for the theory of $1/f$ noise the \tilde{n}_k -component of the mobility fluctuations is of basic interest which can be represented as [4, 9]

$$\tilde{\mu} \equiv \sum_{\mathbf{k}} (\bar{\mu}_k - \bar{\mu}) \tilde{n}_k / \bar{n} V. \quad (1)$$

Here \bar{n} is the statistical average concentration of conduction electrons, V is the semiconductor volume, \mathbf{k} is the quasi-wave vector of a conduction electron and $\bar{\mu}$ is the statistical average mobility of the electron and $\bar{\mu}_k$ is the statistical average component of “local” mobility which is determined by the statistical average quasi-momentum relaxation time $\bar{\tau}_k$ as $\bar{\mu}_k = e\bar{\tau}_k/m$, where m is the electron effective mass. The summing takes place within the range of first Brillouin zone (BZ). Herein and below, the statistical average and the fluctuation components of the corresponding quantities are denoted by the symbols “ $\langle\langle - \rangle\rangle$ ” and “ $\langle\langle \sim \rangle\rangle$ ”, respectively, vector quantities and their magnitudes are denoted by boldface and non-boldface symbols, respectively.

The occupancy fluctuations \tilde{n}_k are expressed by \tilde{f}_k^0 fluctuations of the equilibrium distribution function of conduction electrons f_k^0 as shown below [4, 9]:

$$\tilde{n}_k = -\frac{1}{3} k \frac{d\tilde{f}_k^0}{dk}. \quad (2)$$

Here $f_k^0(t) = \bar{f}_k^0 + \tilde{f}_k^0(t)$, \bar{f}_k^0 is the statistical average distribution function of the equilibrium electron system which is determined by the Fermi–Dirac (or Boltzmann) statistics and $\tilde{f}_k^0(t)$ is time-dependent function which describes random disturbances (fluctuations) of the equilibrium state. By the index “0” of distribution functions it is

emphasized the important fact that fluctuations are equilibrium fluctuations which arise in equilibrium semiconductors, too.

As a result of occupancy fluctuations $\tilde{n}_{\mathbf{k}}$ the equilibrium electron system of the semiconductor transmits to the non-equilibrium state. The $\tilde{n}_{\mathbf{k}}$ and $\tilde{f}_{\mathbf{k}}^0(t)$ functions, which characterize the non-equilibrium state of the electron system, may be represented as a sum of symmetric $\tilde{n}_{\mathbf{k}}^s(t)$, $\tilde{f}_{\mathbf{k}}^{0,s}(t)$ (i.e., $\tilde{n}_{\mathbf{k}}^s(t) = \tilde{n}_{-\mathbf{k}}^s(t)$, $\tilde{f}_{\mathbf{k}}^{0,s} = \tilde{f}_{-\mathbf{k}}^{0,s}$) and asymmetric $\tilde{n}_{\mathbf{k}}^a(t)$, $\tilde{f}_{\mathbf{k}}^{0,a}(t)$ (i.e., $\tilde{n}_{\mathbf{k}}^a(t) = -\tilde{n}_{-\mathbf{k}}^a(t)$, $\tilde{f}_{\mathbf{k}}^{0,a} = -\tilde{f}_{-\mathbf{k}}^{0,a}$) components as shown below:

$$\begin{aligned}\tilde{n}_{\mathbf{k}}(t) &= \tilde{n}_{\mathbf{k}}^s(t) + \tilde{n}_{\mathbf{k}}^a(t), \\ \tilde{f}_{\mathbf{k}}^0(t) &= \tilde{f}_{\mathbf{k}}^{0,s}(t) + \tilde{f}_{\mathbf{k}}^{0,a}(t).\end{aligned}$$

From expression (1) it follows that

$$\tilde{\mu} = \sum_{\mathbf{k}} (\bar{\mu}_{\mathbf{k}} - \bar{\mu}) \tilde{n}_{\mathbf{k}}^s / \bar{n}V + \sum_{\mathbf{k}} (\bar{\mu}_{\mathbf{k}} - \bar{\mu}) \tilde{n}_{\mathbf{k}}^a / \bar{n}V = \sum_{\mathbf{k}} (\bar{\mu}_{\mathbf{k}} - \bar{\mu}) \tilde{n}_{\mathbf{k}}^s / \bar{n}V. \quad (3)$$

Here we have taken into account that summing takes place within the range of Brillouin zone and due to asymmetry of $\tilde{n}_{\mathbf{k}}^a(t)$ the corresponding sum is zero. Therefore, one can state that the electron mobility fluctuations are caused by the symmetric component $\tilde{n}_{\mathbf{k}}^s(t)$ of the occupancy fluctuations $\tilde{n}_{\mathbf{k}}$ [or taking into account relation (2): by the symmetric component of the electron distribution function fluctuations $\tilde{f}_{\mathbf{k}}^{0,s}(t)$]. Moreover, asymmetric component $\tilde{n}_{\mathbf{k}}^a(t)$ or $\tilde{f}_{\mathbf{k}}^{0,a}(t)$ has no contribution to the mobility fluctuations. $\tilde{f}_{\mathbf{k}}^{0,s}(t)$ is a symmetric function, which means that it depends only on electron energy $\varepsilon_{\mathbf{k}}$ (or the magnitude k of the quasi-wave vector \mathbf{k}), and, of course, on the time, too. Therefore, hereafter we can also use the notation $\tilde{f}_{\varepsilon_{\mathbf{k}}}^{0,s}(t)$ for the function $\tilde{f}_{\mathbf{k}}^{0,s}(t)$.

As it is well known [10–12], in the simple case of the semiconductor with a standard band structure the electron quasi-momentum relaxation time $\bar{\tau}_k$ (therefore, “local” mobility $\bar{\mu}_k$ too) depends on electron quasi-wave vector magnitude k by a power-law

$$\begin{aligned}\bar{\tau}_k &= \tau_{0,r} (k/k_T)^{-r}, \\ \bar{\mu}_k &= \frac{e\tau_{0,r}}{m} (k/k_T)^{-r}.\end{aligned} \quad (4a)$$

The statistical average mobility is determined as

$$\bar{\mu} = \frac{e\langle \bar{\tau}_k \rangle}{m} = \frac{e\tau_{0,r}}{m} \Delta_r. \quad (4b)$$

Here $\langle \dots \rangle$ is the symbol of averaging over quantum states of the conduction band (\mathbf{k} -averaging),

$$\Delta_r = \frac{4}{3\sqrt{\pi}} \Gamma\left(\frac{5}{2} - \frac{r}{2}\right),$$

$\Gamma(\dots)$ is the gamma function, $\tau_{0,r}$ is a constant which depends on the electron scattering mechanism and semiconductor parameters [10–12], $k_T = 2mk_B T/\hbar^2$ is the quasi-wave vector of thermal electrons, k_B is the Boltzmann constant, \hbar is the reduced Planck's constant and T is the temperature. The index r characterizes the intraband scattering mechanisms of conduction electrons: with scattering on acoustic phonons $r = 1$, with scattering on ionized impurities $r = -3$, with scattering on neutral impurities $r = 0$, etc. [10–12].

Using expressions (4a) and (4b), Eq. (3) can be written as

$$\tilde{\mu} = \frac{\bar{\mu}}{\bar{n}V\Delta_r} \sum_{\mathbf{k}} \tilde{n}_{\mathbf{k}}^s ((k/k_T)^{-r} - \Delta_r). \quad (5)$$

For further discussions it is more convenient to express the energy of the electron $\varepsilon_{\mathbf{k}} = \hbar^2 k^2/2m$ with the dimensionless quantity x : $x = \varepsilon_{\mathbf{k}}/k_B T$. Then (2) becomes

$$\tilde{n}_{\mathbf{k}} = -\frac{2}{3} x \frac{d\tilde{f}_x^0}{dx}. \quad (6)$$

Using the following transition [11, 12]:

$$\sum_{\mathbf{k}} (\dots) \iff \frac{V}{(2\pi)^3} \int_{\text{BZ}} d\mathbf{k} (\dots), \quad (7)$$

where $d\mathbf{k} = k^2 dk \sin \vartheta d\vartheta d\phi$, then, doing integrations over spherical coordinates ϑ and ϕ , and taking into account the electron spin, (5) can be written as

$$\tilde{\mu} = \frac{\bar{\mu}(2mk_B T/\hbar^2)^{3/2}}{2\pi^2 \bar{n} \Delta_r} \int_0^\infty dx \tilde{n}_{\mathbf{k}}^s \sqrt{x} (x^{-r/2} - \Delta_r), \quad (8)$$

or taking into account expression (6):

$$\tilde{\mu}(t) = -\frac{\bar{\mu}(2mk_B T/\hbar^2)^{3/2}}{3\pi^2 \bar{n} \Delta_r} \int_0^\infty dx \frac{d\tilde{f}_x^{0,s}(t)}{dx} x^{3/2} (x^{-r/2} - \Delta_r). \quad (9)$$

In this work the electron-acoustic phonon scattering is of interest. In this case $r = 1$ [10–12] and

$$\bar{\tau}_k = \frac{\tau_0}{\sqrt{x}}, \quad \bar{\mu}_k = \frac{e\tau_0}{m\sqrt{x}}, \quad \bar{\mu} = \frac{e\langle \bar{\tau}_k \rangle}{m} = \frac{e\tau_0}{m} \frac{4}{3\sqrt{\pi}}, \quad (10)$$

where [11]

$$\tau_0 = \frac{2\pi\hbar^4 v_0^2 \rho}{(2mk_B T)^{3/2} \varepsilon_{\text{def}}^2},$$

ρ is the mass density of the crystal, ε_{def} is the acoustic deformation potential constant and v_0 is the sound velocity of the longitudinal acoustic mode.

One can notice that the intraband scatterings do not change the concentration of the conduction electrons:

$$\tilde{n} = \sum_{\mathbf{k}} \tilde{n}_{\mathbf{k}}^s(t)/V = 0.$$

It means that in Eqs. (5), (8) and (9) the component of the sum or integral which has coefficient Δ_r equals zero. Therefore, Eq. (9) takes the following form:

$$\tilde{\mu}(t) = -\frac{\bar{\mu}(2mk_B T/\hbar^2)^{3/2}}{4\pi^{3/2}\bar{n}} \int_0^\infty dx x \frac{d\tilde{f}_x^{0,s}(t)}{dx}. \quad (11)$$

Doing a simple integration by parts Eq. (11) can be represented as below:

$$\tilde{\mu}(t) = \mu_0 \int_0^\infty dx \tilde{f}_x^{0,s}(t), \quad (12)$$

where

$$\mu_0 = \frac{\bar{\mu}(2mk_B T/\hbar^2)^{3/2}}{4\pi^{3/2}\bar{n}}. \quad (13)$$

From Eq. (12) one can notice that to find out the features of the time dependence of the mobility fluctuations $\tilde{\mu}(t)$, first of all, we need to find out the time dependence of the symmetric component of the electron distribution function fluctuations $\tilde{f}_x^{0,s}(t)$.

3. The Equation of Electron Diffusion in k -Space

Occupancy fluctuations \tilde{n}_k are equilibrium fluctuations [3, 4]. In the state of thermal equilibrium the \mathbf{k} state of the conduction band of the semiconductor is occupied by \bar{n}_k electrons. At a given time instance $t = 0$ electron transmits from one state (energy level) to the conduction band state (energy level) \mathbf{k} or vice versa: from state \mathbf{k} to another state due to random intraband electron–phonon scattering (or generation–recombination transitions). For example, as a result of the electron intraband transition $\mathbf{k}' \rightarrow \mathbf{k}$ the occupancy of the state \mathbf{k} increases and the occupancy of the state \mathbf{k}' decreases. Such transitions lead to the symmetric and asymmetric random changes of the occupancies of conduction band states (or energy levels) or to the fluctuations \tilde{n}_k .

The total quasi-momentum \mathbf{P} and current density \mathbf{J} of the electron system which is described by the distribution function $f_{\mathbf{k}}^0$ are defined as [11]

$$\mathbf{P} \equiv \sum_{\mathbf{k}} \hbar \mathbf{k} f_{\mathbf{k}}^0, \quad (14)$$

$$\mathbf{J} \equiv -\frac{e}{V} \sum_{\mathbf{k}} \frac{\hbar \mathbf{k}}{m} f_{\mathbf{k}}^0 = -\frac{e\mathbf{P}}{mV}. \quad (15)$$

Since

$$f_{\mathbf{k}}^0 = \bar{f}_{\mathbf{k}}^0 + \tilde{f}_{\mathbf{k}}^0 = \bar{f}_{\mathbf{k}}^0 + \tilde{f}_{\mathbf{k}}^{0,a} + \tilde{f}_{\mathbf{k}}^{0,s}, \quad (16)$$

and

$$\sum_{\mathbf{k}} \hbar \mathbf{k} \bar{f}_{\mathbf{k}}^0 = 0, \quad \sum_{\mathbf{k}} \hbar \mathbf{k} \tilde{f}_{\mathbf{k}}^{0,s} = 0,$$

the fluctuations of the total quasi-momentum $\tilde{\mathbf{P}}$ and the current density $\tilde{\mathbf{J}}$ of the electron system of the semiconductor that is not under an external electric field can be written as shown below:

$$\tilde{\mathbf{P}} \equiv \sum_{\mathbf{k}} \hbar \mathbf{k} \tilde{f}_{\mathbf{k}}^{0,a}, \quad (17)$$

$$\tilde{\mathbf{J}} \equiv -\frac{e}{V} \sum_{\mathbf{k}} \frac{\hbar \mathbf{k}}{m} \tilde{f}_{\mathbf{k}}^{0,a} = -\frac{e \tilde{\mathbf{P}}}{mV}. \quad (18)$$

The result is obvious: In a semiconductor which is not under an external electric field there are fluctuating currents $\tilde{\mathbf{J}}$ which are the result of the fluctuations of the total quasi-momentum $\tilde{\mathbf{P}}$ of the electron system. In turn, the quasi-momentum fluctuations are the result of the electron intraband scattering, random generation-recombination transitions or the shot effect [3, 4]. The quasi-momentum fluctuations are described by asymmetric component of the deviation of the electron distribution function $\tilde{f}_{\mathbf{k}}^{0,a}(t)$. In contrast to asymmetric component $\tilde{f}_{\mathbf{k}}^{0,a}$ the contribution of the symmetric component $\tilde{f}_{\mathbf{k}}^{0,s}$ arises under the external electric field as mobility fluctuations [see Eqs. (8), (9) and (12)].

Notice that in the given case the frequency dependence of the spectral density of the current noise is determined by patterns of the decay over time of the initial fluctuation of the distribution function $\tilde{f}_{\mathbf{k}}^0(t=0)$ [7], and therefore by both dependencies $\tilde{f}_{\mathbf{k}}^{0,a}(t)$ and $\tilde{f}_{\mathbf{k}}^{0,s}(t)$. It is well known [11, 12] that in general cases asymmetric deviations $\tilde{f}_{\mathbf{k}}^{0,a}$ decay over time exponentially. This leads to a Lorentzian spectral density of the current noise. But to get a comprehensive knowledge about the full restoration of the electron system equilibrium it is necessary to study and find the time dependence of the symmetric component $\tilde{f}_{\mathbf{k}}^{0,s}(t)$, and, in particular, to find out the laws of decay over time of mobility fluctuations.

Let us discuss this problem on the basis of the following physical model which is of great importance. Consider the symmetric component of the occupancy fluctuations $\tilde{n}_{\mathbf{k}}^s$ which is caused by random electron-acoustic phonon scattering. Assume that at the time instance $t = 0$, the electron absorbs (or emits) acoustic phonon with energy $\hbar\omega_q$ and changes its state of equilibrium and this causes fluctuations $\tilde{n}_{\mathbf{k}}^s(t)$ or $\tilde{f}_{\mathbf{k}}^{0,s}(t)$. At random time instances the electron randomly changes its energy by absorbing or emitting phonons with random energies $\hbar\omega_q$. Therefore, the movement of the electron along the energy axis is a chaotic movement which is known as diffusive movement of an electron along the energy axis [13]. Generally speaking, due to the phonon absorption or emission the equilibrium of both electron and phonon systems

is disturbed. The equilibrium state of the phonon system restores due to the phonon–phonon scattering with the well-known “long-wavelength phonon \rightarrow short-wavelength phonon \rightarrow crystal lattice” mechanism (for more information, see, e.g., Ref. 8). Phonon–phonon scatterings generally are more intense and fast processes. Therefore, in this paper (unlike the problem discussed in Refs. 5, 6) we will suppose that the equilibrium of the phonon system restores very quickly (almost immediately). In other words, it is assumed that the absorption or emission of a phonon by the electron practically does not change the equilibrium of the phonon system [8].

Diffusive movement of the electron along the energy axis is described by the symmetric component of the fluctuations of the electron distribution function $\tilde{f}_x^{0,s}(t)$. In order to determine the dependence $\tilde{f}_x^{0,s}(t)$ one needs to use the Boltzmann kinetic equation which in this case has the following form [11, 12]:

$$\frac{\partial f_{\mathbf{k}}^0(t)}{\partial t} = J_{e\text{-ph}}[g_{\mathbf{q}}^0(t), f_{\mathbf{k}}^0(t)]. \quad (19)$$

Here $J_{e\text{-ph}}$ is the collision integral for electron–phonon scattering, $g_{\mathbf{q}}^0(t)$ is the phonon distribution function, $f_{\mathbf{k}}^0(t)$ is the electron distribution function and \mathbf{q} and \mathbf{k} are the quasi-wave vectors of phonons and electrons, respectively.

For the more probable case of electron–single phonon scattering $J_{e\text{-ph}}$ is represented as [11, 12]

$$J_{e\text{-ph}} = \sum_{\mathbf{q}} w_{\mathbf{q}} \{ [f_{\mathbf{k}+\mathbf{q}}^0(1-f_{\mathbf{k}}^0)(1+g_{\mathbf{q}}^0) - (1-f_{\mathbf{k}+\mathbf{q}}^0)f_{\mathbf{k}}^0g_{\mathbf{q}}^0] \delta(\varepsilon_{\mathbf{k}+\mathbf{q}} - \varepsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{q}}) - [(1-f_{\mathbf{k}-\mathbf{q}}^0)f_{\mathbf{k}}^0(1+g_{\mathbf{q}}^0) - f_{\mathbf{k}-\mathbf{q}}^0(1-f_{\mathbf{k}}^0)g_{\mathbf{q}}^0] \delta(\varepsilon_{\mathbf{k}-\mathbf{q}} - \varepsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{q}}) \}. \quad (20)$$

Here $\varepsilon_{\mathbf{k}}$ is the electron energy in a state which is described by the quasi-wave vector \mathbf{k} and $\hbar\omega_{\mathbf{q}}$ is the acoustic phonon energy, and the factor $w_{\mathbf{q}}$ in the case of electron–acoustic phonon deformation interaction is defined as [10, 11]

$$w_{\mathbf{q}} \equiv \frac{\pi\varepsilon_{\text{def}}^2 q^2}{\rho V \omega_{\mathbf{q}}}.$$

In the case of electron–long-wavelength acoustic phonon interaction when we can use the linear dispersion law $\omega_{\mathbf{q}} = v_0|\mathbf{q}| = v_0q$, for $w_{\mathbf{q}}$ one can write [10–12]

$$w_{\mathbf{q}} = w_0q, \quad w_0 \equiv \frac{\pi\varepsilon_{\text{def}}^2}{\rho v_0 V}. \quad (21)$$

Taking into account the quasi-momentum conservation law ($\mathbf{k}' = \mathbf{k} \pm \mathbf{q}$) Eq. (19) can be represented as

$$\begin{aligned} \frac{df_{\mathbf{k}}^0(t)}{dt} = & \sum_{\mathbf{k}'} w_{\mathbf{k}'-\mathbf{k}} [f_{\mathbf{k}'}^0(1-f_{\mathbf{k}}^0)(g_{\mathbf{k}'-\mathbf{k}}^0+1) - f_{\mathbf{k}}^0(1-f_{\mathbf{k}'}^0)g_{\mathbf{k}'-\mathbf{k}}^0] \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} - \hbar\omega_{\mathbf{k}'-\mathbf{k}}) \\ & - \sum_{\mathbf{k}'} w_{\mathbf{k}-\mathbf{k}'} [f_{\mathbf{k}}^0(1-f_{\mathbf{k}'}^0)(g_{\mathbf{k}-\mathbf{k}'}^0+1) - f_{\mathbf{k}'}^0(1-f_{\mathbf{k}}^0)g_{\mathbf{k}-\mathbf{k}'}^0] \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{k}-\mathbf{k}'}). \end{aligned} \quad (22)$$

Here the peculiarities $\omega_q = v_0|\mathbf{k}' - \mathbf{k}| \equiv \omega_{\mathbf{k}'-\mathbf{k}}$ and $w_q = w_{\mathbf{k}'-\mathbf{k}} = w_0|\mathbf{k}' - \mathbf{k}|$ are used.

According to the above-mentioned model in this paper fluctuations of the phonon system are neglected. It is supposed that the phonon distribution function g_q^0 (or $g_{\mathbf{k}-\mathbf{k}'}^0$) is time-independent quantity and determined by the Bose–Einstein distribution

$$g_{\mathbf{k}-\mathbf{k}'}^0 = \bar{g}_{\mathbf{k}-\mathbf{k}'}^0 = \frac{1}{\exp\left(\frac{\hbar\omega_{\mathbf{k}-\mathbf{k}'}}{k_B T}\right) - 1}. \quad (23)$$

Substituting (16) into Eq. (22) and linearizing it in the case of small deviations ($|\tilde{f}_{\mathbf{k}}^{0,a}|, |\tilde{f}_{\mathbf{k}}^{0,s}| \ll \bar{f}_{\mathbf{k}}^0$), for the function $\tilde{f}_{\mathbf{k}}^{0,s}(t)$ one obtains the following equation:

$$\begin{aligned} \frac{d\tilde{f}_{\mathbf{k}}^{0,s}(t)}{dt} = & \sum_{\mathbf{k}'} w_{\mathbf{k}'-\mathbf{k}} [\tilde{f}_{\mathbf{k}'}^{0,s} (g_{\mathbf{k}'-\mathbf{k}}^0 + 1) - \tilde{f}_{\mathbf{k}}^{0,s} g_{\mathbf{k}'-\mathbf{k}}^0] \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} - \hbar\omega_{\mathbf{k}'-\mathbf{k}}) \\ & - \sum_{\mathbf{k}'} w_{\mathbf{k}-\mathbf{k}'} [\tilde{f}_{\mathbf{k}}^{0,s} (g_{\mathbf{k}-\mathbf{k}'}^0 + 1) - \tilde{f}_{\mathbf{k}'}^{0,s} g_{\mathbf{k}-\mathbf{k}'}^0] \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{k}-\mathbf{k}'}). \end{aligned} \quad (24)$$

Substituting (23) into (24) we have

$$\begin{aligned} \frac{d\tilde{f}_{\mathbf{k}}^{0,s}(t)}{dt} = & \sum_{\mathbf{k}'} w_{\mathbf{k}'-\mathbf{k}} \left[\tilde{f}_{\varepsilon_{\mathbf{k}'}}^{0,s} \frac{e^{\frac{\varepsilon_{\mathbf{k}'-\mathbf{k}}}{k_B T}}}{e^{\frac{\varepsilon_{\mathbf{k}'-\mathbf{k}}}{k_B T}} - 1} - \tilde{f}_{\varepsilon_{\mathbf{k}}}^{0,s} \frac{1}{e^{\frac{\varepsilon_{\mathbf{k}'-\mathbf{k}}}{k_B T}} - 1} \right] \\ & \times [\delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} - \hbar\omega_{\mathbf{k}'-\mathbf{k}}) - \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{k}-\mathbf{k}'})]. \end{aligned} \quad (25)$$

Now let us take into account that the electron-acoustic phonon scattering is quasi-elastic because $\hbar\omega_{\mathbf{k}-\mathbf{k}'} < \varepsilon_{\mathbf{k}}$ [10–12]. Then in expression (25) δ -functions can be represented by Taylor series as

$$\begin{aligned} \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} - \hbar\omega_{\mathbf{k}'-\mathbf{k}}) = & \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}) - \frac{d\delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}})}{d\varepsilon_{\mathbf{k}'}} \hbar\omega_{\mathbf{k}'-\mathbf{k}} \\ & + \frac{1}{2} \frac{d^2\delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}})}{d\varepsilon_{\mathbf{k}'}^2} (\hbar\omega_{\mathbf{k}'-\mathbf{k}})^2 + \dots, \end{aligned} \quad (26)$$

$$\begin{aligned} \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{k}'-\mathbf{k}}) = & \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}}) + \frac{d\delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}})}{d\varepsilon_{\mathbf{k}'}} \hbar\omega_{\mathbf{k}'-\mathbf{k}} \\ & + \frac{1}{2} \frac{d^2\delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}})}{d\varepsilon_{\mathbf{k}'}^2} (\hbar\omega_{\mathbf{k}'-\mathbf{k}})^2 + \dots. \end{aligned} \quad (27)$$

From series (26) and (27) it follows that

$$\delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} - \hbar\omega_{\mathbf{k}'-\mathbf{k}}) - \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{k}'-\mathbf{k}}) = -2 \frac{d\delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}})}{d\varepsilon_{\mathbf{k}'}} \hbar\omega_{\mathbf{k}'-\mathbf{k}} + \dots. \quad (28)$$

Confining to the first term of series (28), from Eq. (25) one obtains

$$\frac{d\tilde{f}_{\varepsilon_{\mathbf{k}}}^{0,s}(t)}{dt} = -2 \sum_{\mathbf{k}'} w_{\mathbf{k}'-\mathbf{k}} \left[\tilde{f}_{\varepsilon_{\mathbf{k}'}}^{0,s} \frac{e^{\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_B T}}}{e^{\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_B T}} - 1} - \tilde{f}_{\varepsilon_{\mathbf{k}}}^{0,s} \frac{1}{e^{\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_B T}} - 1} \right] \frac{d\delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}})}{d\varepsilon_{\mathbf{k}'}} \hbar \omega_{\mathbf{k}'-\mathbf{k}}. \quad (29)$$

Using dimensionless quantity x and Eq. (21) (or the relation $w_{\mathbf{q}} = w_{\mathbf{k}'-\mathbf{k}} = w_0 |\mathbf{k}' - \mathbf{k}|$), (29) can be represented as shown below:

$$\frac{d\tilde{f}_x^{0,s}(t)}{dt} = -\frac{2w_0}{(k_B T)^2} \sum_{\mathbf{k}'} |\mathbf{k}' - \mathbf{k}| \left[\tilde{f}_{x'}^{0,s} \frac{e^{x'-x}}{e^{x'-x} - 1} - \tilde{f}_x^{0,s} \frac{1}{e^{x'-x} - 1} \right] \frac{d\delta(x' - x)}{dx'} \hbar \omega_{\mathbf{k}'-\mathbf{k}}. \quad (30)$$

Taking into account that the dispersion law of long-wavelength phonons is linear, $\omega_{\mathbf{k}'-\mathbf{k}} = v_0 |\mathbf{k}' - \mathbf{k}|$, from (30) one obtains

$$\frac{d\tilde{f}_x^{0,s}(t)}{dt} = -\frac{2w_0 \hbar v_0}{(k_B T)^2} \sum_{\mathbf{k}'} |\mathbf{k}' - \mathbf{k}|^2 \left[\tilde{f}_{x'}^{0,s} \frac{e^{x'-x}}{e^{x'-x} - 1} - \tilde{f}_x^{0,s} \frac{1}{e^{x'-x} - 1} \right] \frac{d\delta(x' - x)}{dx'}. \quad (31)$$

Using transition (7) and integrating over angles ϑ and ϕ , (31) can be represented as shown below:

$$\begin{aligned} \frac{d\tilde{f}_x^{0,s}(t)}{dt} = & -\frac{2V(k_B T)^{1/2} w_0 \hbar v_0 (2m/\hbar^2)^{5/2}}{(2\pi)^2} \\ & \times \int_0^\infty dx' \sqrt{x'} (x' + x) \left[\tilde{f}_{x'}^{0,s} \frac{e^{x'-x}}{e^{x'-x} - 1} - \tilde{f}_x^{0,s} \frac{1}{e^{x'-x} - 1} \right] \frac{d\delta(x' - x)}{dx'}. \end{aligned} \quad (32)$$

Using the δ -integration and expression (21) for w_0 , from Eq. (32) one obtains

$$\begin{aligned} \frac{d\tilde{f}_x^{0,s}(t)}{dt} = & \frac{2\pi \varepsilon_{def}^2 (k_B T)^{1/2} (2m/\hbar^2)^{5/2} \hbar}{(2\pi)^2 \rho} \\ & \times \frac{d}{dx'} \left\{ (x'^{3/2} + x\sqrt{x'}) \left[\tilde{f}_{x'}^{0,s} \frac{e^{x'-x}}{e^{x'-x} - 1} - \tilde{f}_x^{0,s} \frac{1}{e^{x'-x} - 1} \right] \right\} \Big|_{x'=x}. \end{aligned} \quad (33)$$

It is not difficult to see that the right-hand side of Eq. (33) is a 0/0-type uncertainty. To calculate it, we need to use the L'Hospital's rule. As a result, we obtain the following equation:

$$\frac{d\tilde{f}_x^{0,s}(t)}{dt} = \frac{1}{\tau_M} \left[x^{3/2} \frac{d^2 \tilde{f}_x^{0,s}}{dx^2} + (x^{3/2} + 2\sqrt{x}) \frac{d\tilde{f}_x^{0,s}}{dx} + 2\sqrt{x} \tilde{f}_x^{0,s} \right], \quad (34)$$

where

$$\frac{1}{\tau_M} \equiv \frac{2\pi \varepsilon_{def}^2 (k_B T)^{1/2} \hbar (2m/\hbar^2)^{5/2}}{(2\pi)^2 \rho}.$$

Equation of the form (34) is known as the Fokker–Planck equation in \mathbf{k} -space [14]. Equation (34) describes the chaotic movement of the electron along the energy axis. The diffusion coefficient $D_x = x^{3/2}/\tau_M$ is not a constant quantity, it depends on electron energy x .

Solving Eq. (34) and defining the function $\tilde{f}_{\mathbf{k}}^{0,s}(t)$ it will be possible to find out time dependencies of fluctuations of both electron lattice mobility [see Eq. (12)] and total energy of electron system $\tilde{E}(t)$:

$$\tilde{E}(t) = \frac{2V}{(2\pi)^3} \int_{\text{BZ}} d\mathbf{k} \varepsilon_{\mathbf{k}} \tilde{f}_x^{0,s}(t).$$

It is important to note that Eq. (34) and its solution must satisfy the condition $d\tilde{n}(t)/dt = 0$. Indeed, Eq. (34) was obtained on the basis of intraband electron–phonon scattering. Intraband scattering cannot change the concentration of conduction electrons n . It must be a time-independent constant quantity:

$$\frac{dn}{dt} = \frac{d(\bar{n} + \tilde{n})}{dt} = \frac{d\tilde{n}}{dt} = 0.$$

Now, let us show that the function determined by Eq. (34) truly satisfies the above-given condition. For this purpose we use the well-known relation [11, 12] between conduction electron concentration and electron distribution function:

$$n = \frac{2}{(2\pi)^3} \int_{\text{BZ}} d\mathbf{k} f_{\mathbf{k}}^0,$$

where integration is carried out within the first Brillouin zone. Therefore

$$\tilde{n}(t) = \frac{2}{(2\pi)^3} \int_{\text{BZ}} d\mathbf{k} \tilde{f}_x^{0,s}(t).$$

Passing from integration over $d\mathbf{k}$ to integration over dx , $\tilde{n}(t)$ can be written as

$$\tilde{n}(t) = b \int_0^\infty dx \sqrt{x} \tilde{f}_x^{0,s}, \quad (35)$$

where

$$b \equiv \frac{(2mk_B T/\hbar^2)^{3/2}}{2\pi^2}.$$

Taking into account relation (35), multiplying Eq. (34) by $b\sqrt{x}dx$ and making integration over dx within the range $[0, \infty)$, one obtains

$$\frac{d\tilde{n}(t)}{dt} = \frac{b}{\tau_M} \int_0^\infty dx \sqrt{x} \left[x^{3/2} \frac{d^2 \tilde{f}_x^{0,s}}{dx^2} + (x^{3/2} + 2\sqrt{x}) \frac{d\tilde{f}_x^{0,s}}{dx} + 2\sqrt{x} \tilde{f}_x^{0,s} \right]. \quad (36)$$

Using transformations of integration by parts it is not difficult to make sure that the right-hand side of Eq. (36) is equal to zero. So, we obtain

$$\frac{d\tilde{n}(t)}{dt} = \frac{d}{dt} \left(b \int_0^\infty dx \sqrt{x} \tilde{f}_x^{0,s} \right) = 0. \quad (37)$$

This means that Eq. (34) satisfies the condition $d\tilde{n}(t)/dt = 0$ [or $n(t) = \text{const.}$].

4. Discussion and Conclusions

Note that some general and important issues related to electron distribution function, quasi-momentum and energy fluctuations were discussed and summarized in various works (see, e.g., Refs. 15–18). At the same time electron mobility fluctuations (fluctuation sources, decay peculiarities, statistical characteristics and so on) in equilibrium and non-equilibrium, intrinsic and extrinsic semiconductors practically are not investigated in detail. Below the electron lattice mobility fluctuations, their origin and decay peculiarities are discussed.

From expression (12) one can notice that to determine electron lattice mobility fluctuations caused by electron-acoustic phonon scattering and to find out patterns of time dependence $\tilde{\mu}(t)$, first of all, one needs to solve Eq. (34). But the analysis of literature data on linear second-order partial differential equations in two variables (see, e.g., Refs. 19–21) shows that analytic solutions of equations of parabolic type (34) practically are not investigated. In the given case it is possible to find only either numerical solutions or approximate analytic solutions of Eq. (34) based on the known approximate methods [21]. But before passing on to approximate solutions, first of all, by using Eq. (34) we should try to find another equation for $\tilde{\mu}(t)$. To do this, let us differentiate Eq. (12) with respect to time:

$$\frac{d\tilde{\mu}(t)}{dt} = \mu_0 \int_0^\infty dx \frac{d\tilde{f}_x^{0,s}(t)}{dt}, \quad (38)$$

then, substitute $d\tilde{f}_x^{0,s}/dt$ from Eq. (34) into the right-hand side of Eq. (38). After simple integration by parts one obtains the following equation:

$$\frac{d\tilde{\mu}(t)}{dt} = \frac{\mu_0}{\tau_M} \int_0^\infty dx \left(-\frac{1}{4\sqrt{x}} + \frac{1}{2}\sqrt{x} \right) \tilde{f}_x^{0,s}. \quad (39)$$

Now, differentiating Eq. (39) over time t one obtains

$$\frac{d^2\tilde{\mu}(t)}{dt^2} = -\frac{\mu_0}{4\tau_M} \int_0^\infty dx \frac{d}{dt} \left(\frac{\tilde{f}_x^{0,s}}{\sqrt{x}} \right) + \frac{\mu_0}{2\tau_M} \int_0^\infty dx \frac{d}{dt} \left(\sqrt{x} \tilde{f}_x^{0,s} \right). \quad (40)$$

The second term on the right-hand side of this equation is equal to zero [see Eq. (37)]. One can calculate the first integral on the right-hand side of Eq. (40) using Eq. (34) again. For this, let us multiply Eq. (34) by dx/\sqrt{x} and integrate over dx in the range $[0, \infty)$. Then, integrating by parts and taking into account Eq. (12),

we obtain

$$\frac{d}{dt} \int_0^\infty dx \left(\frac{\tilde{f}_x^0}{\sqrt{x}} \right) = \frac{1}{\tau_M} \int_0^\infty dx \tilde{f}_x^0 = \frac{1}{\tau_M} \frac{\tilde{\mu}}{\mu_0}. \quad (41)$$

Substituting Eq. (41) into Eq. (40), for electron lattice mobility fluctuations one obtains the following equation:

$$\frac{d^2 \tilde{\mu}(t)}{dt^2} = -\frac{1}{4\tau_M^2} \tilde{\mu}(t). \quad (42)$$

As it is seen this equation is simple and to find out the $\tilde{\mu}(t)$ dependence it is not needed to return to the solution of (34). The solution of Eq. (42) is a stochastic harmonic function

$$\tilde{\mu}(t) = A \cos(t/2\tau_M + \varphi), \quad (43)$$

where $1/2\tau_M = \omega$ is the vibration frequency, and the amplitude A and the initial phase φ are random quantities.

Processes described by an equation of type (43) are known as harmonic oscillations with random parameters (amplitude A and initial phase φ).

As can be seen from the solution (43), the mobility fluctuation does not decay over time, it varies harmoniously with the characteristic frequency $\omega = 1/2\tau_M$. Using Eq. (10) the characteristic time τ_M can be represented in a more convenient form for numerical estimations:

$$\frac{1}{\tau_M} = \frac{8}{3\sqrt{\pi}} \frac{mv_0^2}{k_B T} \frac{1}{\bar{\tau}},$$

where $\bar{\tau} \equiv \langle \bar{\tau}_k \rangle$ is the electron mean-free time due to electron-acoustic phonon scattering. In particular, for Si at room temperature $v_0 \approx (8.4 - 9.4) \cdot 10^5$ cm/s, $m \approx 0.3m_0$ [22] and one obtains following estimation: $\tau_M \approx 120\bar{\tau}$. The period of mobility oscillations $T = 2\pi/\omega = 4\pi\tau_M \approx 1.5 \cdot 10^3\bar{\tau}$ is approximately three orders of magnitude longer than the electron mean-free time $\bar{\tau}$.

If we take into account the expression for electron energy relaxation time τ_ε given in Ref. 23, then the characteristic time τ_M can be represented also as

$$\frac{1}{\tau_M} = \frac{3\sqrt{\pi}}{8} \frac{1}{\tau_\varepsilon}.$$

It can be stated that the time τ_M practically coincides with energy relaxation time τ_ε . Despite this circumstance, the connection between electron lattice mobility and energy fluctuations is not of general character. As an example the case of electron-electron scattering can be considered. Suppose that in the initial state two conduction electrons are in the states with energies ε_1 (or $x_1 = \varepsilon_1/k_B T$) and ε_2 (or $x_2 = \varepsilon_2/k_B T$), and have lattice local mobilities $\mu_{k_1} = e\tau_0/m\sqrt{x_1}$ and $\mu_{k_2} = e\tau_0/m\sqrt{x_2}$, respectively. In that case the \mathbf{k} -averaged lattice mobility of the system of two electrons is determined as $\mu = (e\tau_0/m\sqrt{x_1} + e\tau_0/m\sqrt{x_2})/2$. Energies

and local mobilities of electrons change as a result of electron–electron scattering. One of these electrons transits to the new state x'_1 , while the other one transits to the state x'_2 , with local mobilities $\mu'_{k'_1} = e\tau_0/m\sqrt{x'_1}$ and $\mu'_{k'_2} = e\tau_0/m\sqrt{x'_2}$, respectively. Due to the scattering the total energy of the above-mentioned electron system does not change ($x_1 + x_2 = x'_1 + x'_2$) while \mathbf{k} -averaged local mobility does change: $\mu' = (e\tau_0/m\sqrt{x'_1} + e\tau_0/m\sqrt{x'_2})/2$. It is obvious that $\mu \neq \mu'$. The above-mentioned statement can be generalized for the whole electron system, too. In this example the total energy of the electron system does not fluctuate but the lattice mobility does: $\tilde{\mu} = \mu' - \mu \neq 0$.

The inelastic character of the electron–acoustic phonon scattering lies on the basis of above-mentioned features of electron lattice mobility fluctuations. Assuming that the scattering is elastic and neglecting the energy $\hbar\omega_q$ in the argument of δ -function on the right-hand side of Eq. (23) one can obtain $d\tilde{f}_k^{0,s}(t)/dt = 0$, which means that the initial fluctuation does not decay over time. What is said above corresponds to the following fact: In the case of strongly elastic scattering the $\mathbf{k} \rightarrow \mathbf{k}'$ and $\mathbf{k}' \rightarrow \mathbf{k}$ electron transitions occur only on a constant-energy surface (the total number of electrons does not change on the given surface) and if at the initial time an electron occupancy of such a surface differs from the equilibrium value, then no equilibrium state can be established or restored [13].

In the case considered in this paper, the initial fluctuation of occupancy of the constant-energy surface and the features of the time dependence of the fluctuation are related to electron–phonon scatterings. It can be assumed that the initial distribution of electrons differs from the equilibrium distribution only for quasi-discrete states \mathbf{k}_0 and \mathbf{k}'_0 so that the condition $\tilde{n} = 0$ takes place. In other words, the distribution function is perturbed only in the vicinity of points \mathbf{k}_0 and \mathbf{k}'_0 (Fig. 1).

The future behavior of $\tilde{f}_k^{0,s}(t = 0)$ is determined by the kinematics of electron transitions. Due to the energy conservation law an electron in the state \mathbf{k} (it is assumed that $k \geq mv_0/\hbar$ [13]) can absorb phonons with energy $0 \leq \hbar\omega_q \leq \hbar\omega_a$ (where $\hbar\omega_a = 2\hbar kv_0 + 2mv_0^2$) and emit phonons with energy $0 \leq \hbar\omega_q \leq \hbar\omega_e$

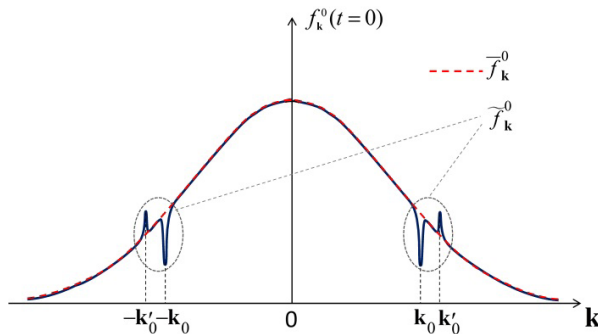


Fig. 1. Symmetric component of the initial fluctuation of electron distribution function due to electron–phonon random scattering.

(where $\hbar\omega_e = 2\hbar kv_0 - 2mv_0^2$). The relation (43) for time dependence of the lattice mobility fluctuations apparently is in agreement and is directly connected to the difference of the energy intervals of absorbed and emitted phonons ($\hbar\omega_e \neq \hbar\omega_a$) and to the consequences of this difference. Moreover, electron transitions with phonon absorption increase the value of negative lattice mobility fluctuations and electron transitions with phonon emission increase the value of positive lattice mobility fluctuations.

Equation (34) and therefore Eq. (42) were obtained by linearization of the Boltzmann equation, when second-order non-linear terms were neglected. We get a very important result: in the linearization approximation equilibrium of the electron system does not restore. It is obvious that for the decay of electron mobility fluctuations and for restoration of equilibrium of the electron system we need to refuse the linearization approximation and take into account non-linear terms. In that case it can be expected that mobility fluctuations will decay over time not exponentially but by a more slower power-law.

The main conclusions of the paper are as follows:

- (i) In the framework of linearization approximation of the Boltzmann fluctuation equation electron lattice mobility fluctuation is not a damping quantity.
- (ii) Time dependence of an electron lattice mobility fluctuation is described by stochastic harmonic function, where amplitude and initial phase are random quantities.

References

- [1] T. Kang, Optimization of signal-to-noise ratio in semiconductor sensors via on-chip signal amplification and interface-induced noise suppression, Ph.D. thesis, University of Michigan (2015).
- [2] M. D. Hammig, T. Kang, M. Jeong and M. Jarrett, Suppression of interface-induced noise by the control of electron-phonon interactions, *IEEE Trans. Nucl. Sci.* **60**(4) (2013) 2831–2839.
- [3] S. V. Melkonyan, F. V. Gasparyan and H. V. Asriyan, Main sources of electron mobility fluctuations in semiconductors, *Proc. SPIE* **6600** (2007) 66001K.
- [4] S. V. Melkonyan, Non-Gaussian conductivity fluctuations in semiconductors, *Physica B* **405**(1) (2010) 379–385.
- [5] S. V. Melkonyan, V. M. Aroutiounian, F. V. Gasparyan and H. V. Asriyan, Phonon mechanism of mobility equilibrium fluctuation and properties of 1/f-noise, *Physica B* **382**(1–2) (2006) 65–70.
- [6] S. V. Melkonyan, V. M. Aroutiounyan, F. V. Gasparyan and C. E. Korman, Peculiarities of electron distribution function's fluctuations damping in homogenous semiconductors, *Physica B* **357**(3–4) (2005) 398–407.
- [7] S. V. Melkonyan, On the low-frequency limit of the Schönfeld pulse 1/f-law, *Physica B* **403**(12) (2008) 2029–2035.
- [8] V. L. Gurevich, *Kinetics of Phonon Systems* (Nauka, Moscow, 1980).
- [9] S. V. Melkonyan, H. V. Asriyan, Ash. V. Surmalyan and J. M. Smulko, Electron mobility variance in semiconductors: The variance approach, *Armen. J. Phys.* **4**(1) (2011) 62–73.
- [10] C. Hamaguchi, *Basic Semiconductor Physics* (Springer, 2010).

- [11] B. M. Askerov, *Electron Transport Phenomena in Semiconductors* (World Scientific, 1994).
- [12] A. I. Anselm, *Introduction to Semiconductor Theory* (Prentice Hall, 1982).
- [13] V. F. Gantmakher and I. B. Levinson, *Carrier Scattering in Metals and Semiconductors* (Elsevier Science, 1987).
- [14] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics*, Vol. 10 (Butterworth-Heinemann, Oxford, 1999).
- [15] S. V. Gantsevitch, V. L. Gurevich and R. Katilius, Theory of fluctuations in non-equilibrium electron gas, *Riv. Nuovo Cimento* **2**(5) (1979) 1–88.
- [16] I. M. Dikman and P. M. Tomchuk, *Transport and Fluctuations Phenomena in Semiconductors* (Naukova Dumka, Kiev, 1981).
- [17] H. Hartnagel, R. Katilius and A. Matulionis, *Microwave Noise in Semiconductor Devices* (Wiley, New York, 2001).
- [18] N. A. Zakhlenyuk, V. A. Kochelap and V. V. Mitin, Non-equilibrium fluctuations in semiconductors associated with energy scattering of charge carriers by lattice optical phonons, *Sov. Phys. -JETP* **68**(4) (1989) 863-872.
- [19] A. D. Polyinin and V. E. Nazaikinskii, *Handbook of Linear Partial Differential Equations for Engineers and Scientists* (Chapman and Hall/CRC Press, Boca Raton, 2016).
- [20] A. D. Polyinin and V. F. Zaitsev, *Handbook of Exact Solutions for Ordinary Differential Equations* (Chapman and Hall/CRC Press, Boca Raton, 2003).
- [21] E. Kamke, *Differentialgleichungen Lösungsmethoden und Lösungen* (Springer, 1977).
- [22] A. Dargys and J. Kundrotas, *Handbook on Physical Properties of Ge, Si, GaAs and InP* (Science and Encyclopedia Publishers, Vilnius, 1994).
- [23] K. Zeeger, *Semiconductor Physics* (Springer, 2004).