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# GENERALIZED LOVE WAVES IN BI-MATERIAL WAVEGUIDE WITH VISCOUS SLIP INTERFACE

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The purpose of this study is to investigate shear SH wave travelling in a finite-width waveguide consisting of two elastic sub-layers imperfectly bonded at viscous linear slip interface where velocity of displacement discontinuity are taken to be linearly related to stress traction which is continuous across the interface. The SH waves in finite-width layered waveguide with perfect contact conditions of continuity of displacement and stress interface is considered in [1]. Existence conditions and character of the propagation of viscoelastic shear surface waves in an elastic half-space with a viscoelastic coating are considered in [2].

Let consider in Cartesian system $\left(-a_2 < x < a_1, -\infty < y < \infty, -\infty < z < \infty\right)$ a bi-material waveguide made from different elastic materials $A(0 < x < a_1)$ and $B(-a_2 < x < 0)$ and imperfectly bonded at $x = 0$. The equation of motion for SH wave is given by

$$\sigma_{xz}^{(s)} = G^{(s)} \frac{\partial u_z^{(s)}}{\partial x}; \sigma_{yz}^{(s)} = G^{(s)} \frac{\partial u_z^{(s)}}{\partial y}, \quad \frac{\partial \sigma_{xz}^{(s)}}{\partial x} + \frac{\partial \sigma_{yz}^{(s)}}{\partial y} = \rho^{(s)} \frac{\partial^2 u_z^{(s)}}{\partial t^2} \qquad (1)$$

where $u_z^{(s)}$ are elastic displacements, $\sigma_{xz}^{(s)}, \sigma_{yz}^{(s)}$ are the shear stresses, $\rho^{(s)}$ are the mass densities, $G^{(s)}$ are the shear elastic modulus, respectively. The indexes $s = 1, s = 2$ stand for two sub-layers $A$ and $B$, respectively.

A model of an imperfectly bonded linear viscous slip interface between two elastic sublayers will be used [3]. According to this model the following contact conditions are valid at slip interface $x = 0$

$$\sigma_{xz}^{(1)}(0, y, t) = \sigma_{xz}^{(2)}(0, y, t); \frac{\partial}{\partial t}\left(u_z^{(1)}(0, y, t)\right) - \frac{\partial}{\partial t}\left(u_z^{(2)}(0, y, t)\right) = \eta \sigma_{xz}^{(1)}(0, y, t). \qquad (2)$$

We assume solutions in form of the plane time-harmonic wave travelling along the $y$-direction, $u_z^{(s)}(x, y, t) = u^{(s)}(x) \exp\left[i(ky - \omega t)\right]$, where $\omega$ is the wave angular frequency, $k$ is the wave number.

Since the interface conditions at $x=0$ will be imposed on functions $u^{(s)}(x), \sigma_{xz}^{(s)}(x)$ it is convenient to introduce the column vectors

$$U^{(s)}(x)=\left(u^{(s)}(x), \sigma_{xz}^{(s)}(x)\right)^T .$$

In matrix form the solutions of (1) and interface conditions (2) can be cast as

$$U^{(s)}(x)=\mathrm{F}^{(s)}(x)\cdot C^{(s)}, \quad U^{(1)}(0)=\mathrm{S}U^{(2)}(0), \tag{3}$$

where column vectors $C^{(s)}=\left(c_1^{(s)}, c_2^{(s)}\right)^T$ are constants,

$$F^{(s)}(x)=\begin{pmatrix} \exp(iq_s x), & \exp(-iq_s x) \\ iG_s q_s \exp(iq_s x), & -iG_s q_s \exp(-iq_s x) \end{pmatrix}, \; S=\begin{pmatrix} 1 & i\eta\omega^{-1} \\ 0 & 1 \end{pmatrix},$$

$q_s=\sqrt{\omega^2/\mathrm{c}_{ts}^2-k^2}$; $\mathrm{c}_{ts}=\sqrt{G_s/\varrho_s}$ are the velocities of shear elastic waves. Eliminating vectors $C_s$ in (3), the relations linking $U^{(s)}(x)$ vector values at the boundaries of each sub-layer via transfer matrix can be found as

$$U^{(1)}(a_1)=T_1 U^{(1)}(0); U^{(2)}(0)=T_2 U^{(2)}(-a_2),$$

$$T_s=\begin{pmatrix} \cos(a_s q_s) & (q_s G_s)^{-1}\sin(a_s q_s) \\ -q_s G_s \sin(a_s q_s) & \cos(a_s q_s) \end{pmatrix}.$$

Using the slip interface condition at interface $x=0$, we come to the following relation which links the vectors at the waveguide walls $x=a_1, x=-a_2$

$$U^{(1)}(a_1)=MU^{(2)}(-a_2); \; M=T_1 S T_2 .$$

By means of this $4\times 4$ matrix it is easy to obtain the dispersion equations for different boundary conditions imposed at waveguide walls. The traction free boundary conditions at the waveguide walls are considered

$$\sigma_{xz}^{(1)}(a_1,y,t)=0; \; \sigma_{xz}^{(2)}(-a_2,y,t)=0 .$$

The detailed numerical analysis of dispersion equation is carried out. Effects of interface viscosity properties on dynamic process in bi-material waveguide are studied. The corresponding Love surface wave multipole evanescent modes are studied. The results demonstrate the significant effects of interface viscous compliance on elastic frequencies.


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2. *Belubekyan V., Sarkisyan S.* Love problem for a half-space with a viscoelastic coating // NAS RA Repors. – 2018. – **118** (1) (In Russian).
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